Multicomponent FQHE

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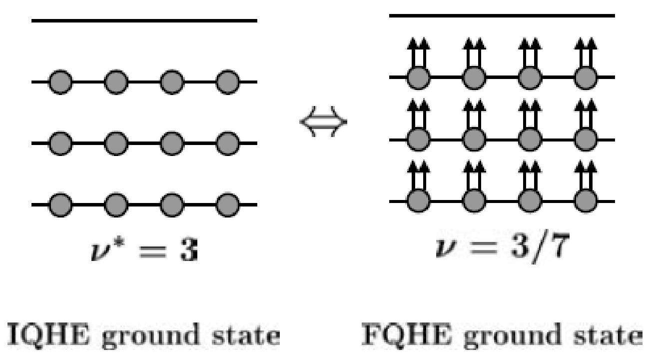
Last time: composite fermions

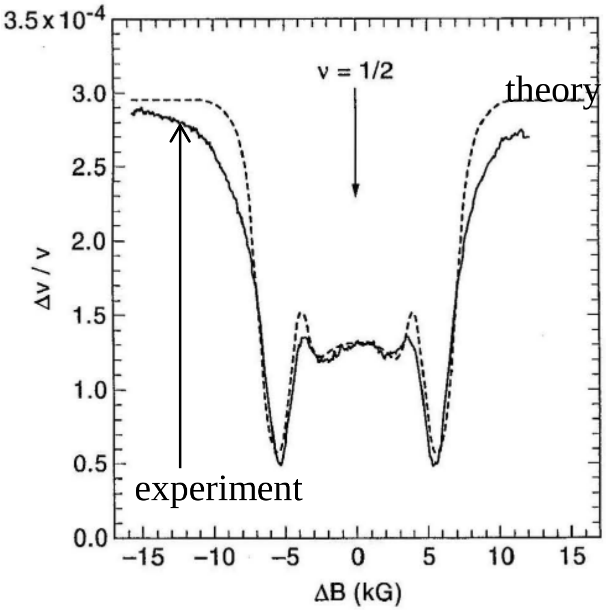
- **Review of FQHE**
  - Laughlin states $\nu = 1/m$

$$\Psi_{1/m}(\{z_i\}) = \prod_{j<k}^{N} \left(\frac{z_j - z_k}{\ell_B}\right)^m \exp\left\{-\frac{1}{4\ell_B^2}\sum_{\ell=1}^{N}|z_\ell|^2\right\}$$

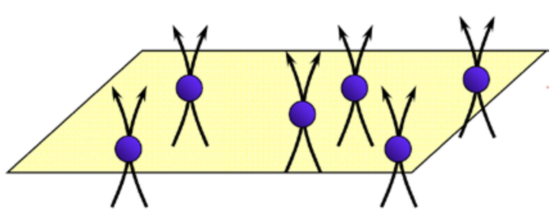
  - Haldane's hierarchical construction
  - Composite fermion "trick"

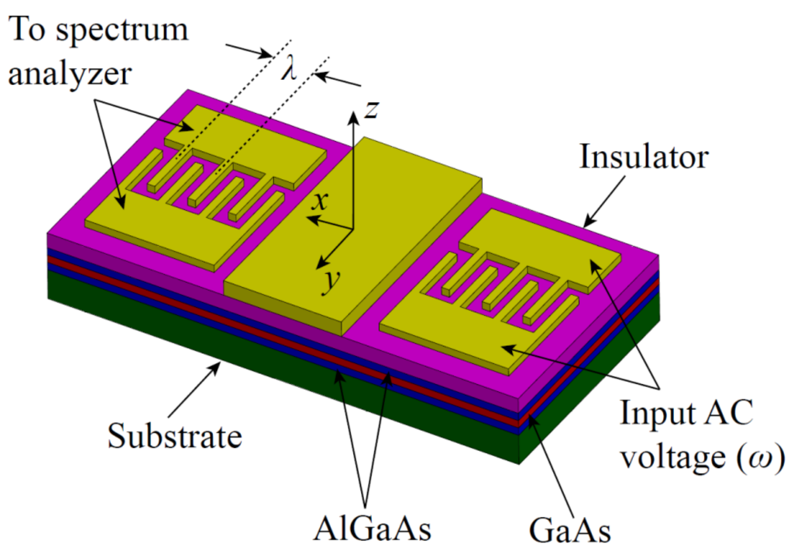


$$\nu = \frac{\nu^*}{2p\nu^* \pm 1}$$



  - HLR: Composite fermion "theory"



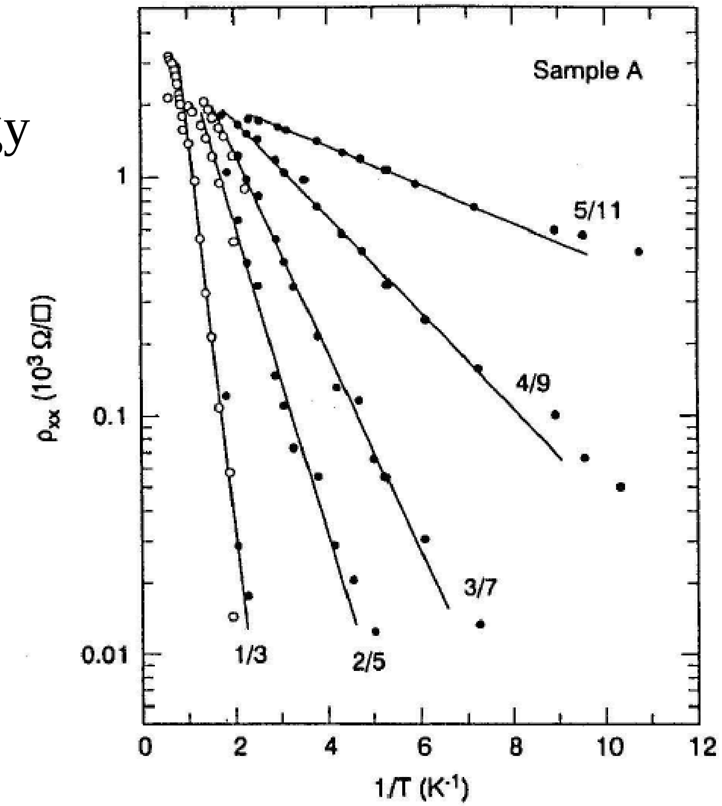


Review of FQHE experimental techniques

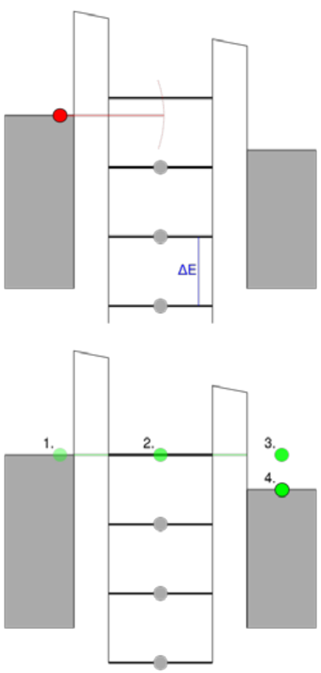
- **Measuring bulk energy gap**
  - Arrhenius-type thermal activation energy

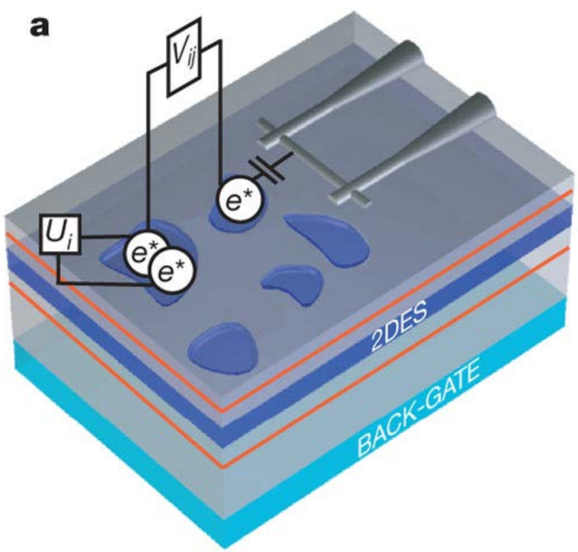
$$\rho_{xx} \propto \exp\left[-\frac{\Delta_{\mathrm{total}}}{k_{\mathrm{B}}T}\right]$$

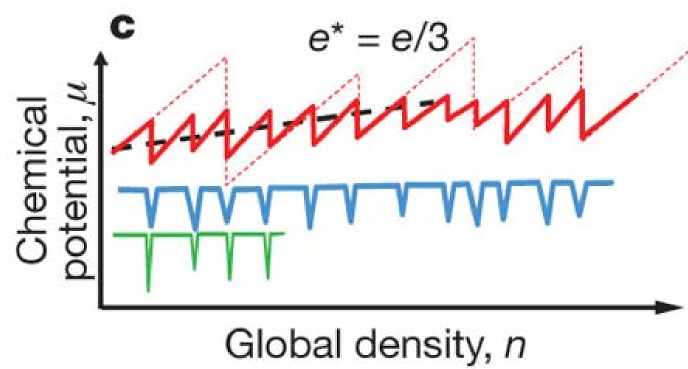
  - Orbital and spin components included in Δ_{total}



- **Local compressibility measurements**

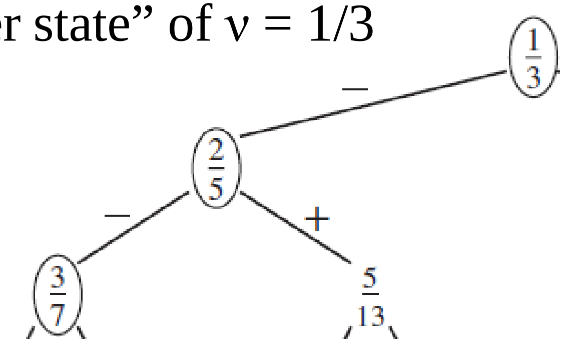


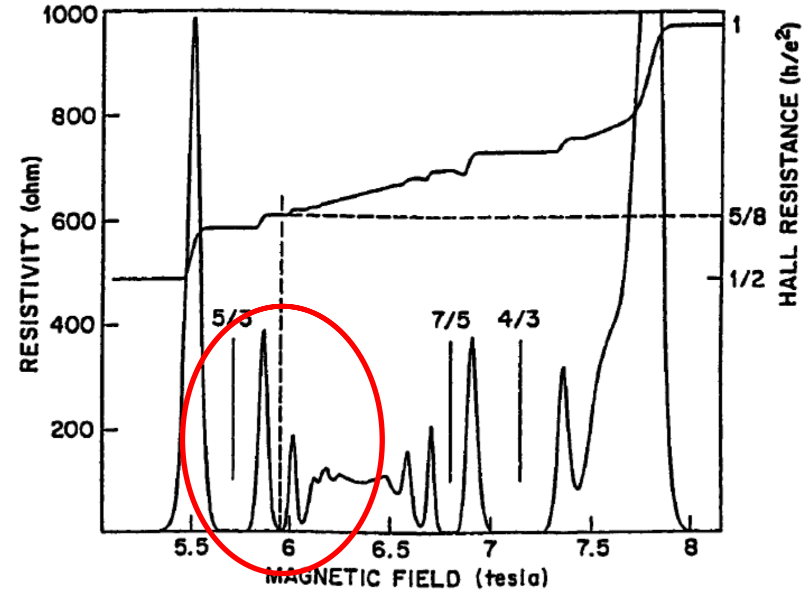




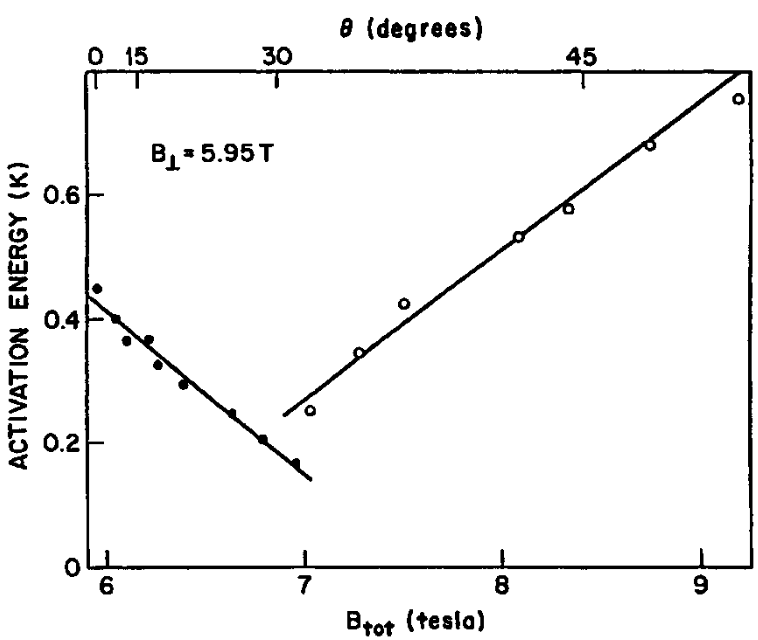
The case of ν = 2/5

- **Tilted Field Technique**
  - "Daughter state" of ν = 1/3

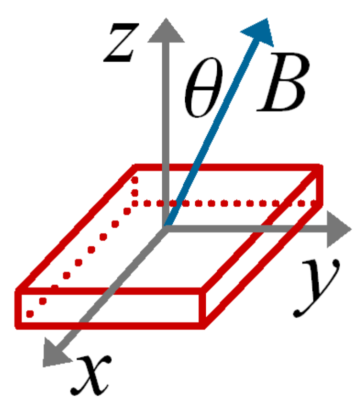


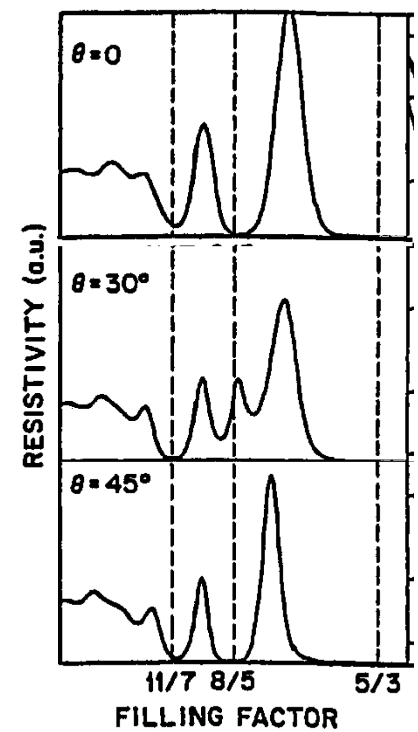


  - Test effect of **in-plane** B-field in particle-hole conjugate state ν = 8/5 (2 – 2/5)



$$\Delta_{\text{total}} = \Delta_{\text{orbital}} - |g|\mu_{\text{B}} B_{\text{tot}} \Delta S$$





Accuracy of the spinless approximation

- **Spin DOF in FQHE in GaAs**
  - Explicit Laughlin wavefunction

$$\Phi_0 = \frac{h}{e} \approx 4 \times 10^{-11}\ \mathrm{T}\cdot\mathrm{cm}^2$$

$$\Psi_{1/m}(\{z_i\}) = \prod_{j<k}^{N} \left(\frac{z_j - z_k}{\ell_\mathrm{B}}\right)^m \exp\left\{-\frac{1}{4\ell_\mathrm{B}^2}\sum_{\ell=1}^{N} |z_\ell|^2\right\} |\uparrow\uparrow \ldots \uparrow\rangle$$

$$\boxed{\frac{B}{\Phi_0} = m\rho_\mathrm{e}}$$

  - Halperin points out inconvenient detail

$$E_\mathrm{Z} = 2g\mu_\mathrm{B}\mathbf{B}\cdot\mathbf{S} = \frac{g}{2}\left(\frac{m_\mathrm{b}}{m_\mathrm{e}}\right)\hbar\omega_\mathrm{c} \approx 0.3\,B[\mathrm{T}]\ \mathrm{K} \qquad V_\mathrm{C} \equiv \frac{e^2}{\epsilon\ell_\mathrm{B}} \approx 50\sqrt{B[\mathrm{T}]}\ \mathrm{K}$$

  - Example: $\nu = 1/3$ Laughlin state

$$\hbar\omega_\mathrm{c} = \hbar\frac{eB}{m_\mathrm{b}c} \approx 20\,B[\mathrm{T}]\ \mathrm{K}$$

$$\rho_\mathrm{e} = \frac{4}{3}\times 10^{11}\ \mathrm{cm}^{-2}$$

$$\begin{aligned} B &= m\rho_\mathrm{e}\Phi_0 \\ &= 3 \times \left(\frac{4}{3}\times 10^{11}\ \mathrm{cm}^{-2}\right) \times (4\times 10^{11}\ \mathrm{T}\cdot\mathrm{cm}^2) \\ &= 16\ \mathrm{T} \end{aligned}$$

  - Compare energy scales for $\nu = 1/3$ Laughlin state

$$\begin{aligned} E_\mathrm{Z} &= 0.3\,B(T)\ \mathrm{K} \\ &\approx 5\ \mathrm{K} \end{aligned} \qquad \begin{aligned} V_\mathrm{C} &= 50\sqrt{B(T)}\ \mathrm{K} \\ &= 200\ \mathrm{K} \end{aligned} \qquad \begin{aligned} \hbar\omega_\mathrm{c} &= 20\,B(T)\ \mathrm{K} \\ &= 320\ \mathrm{K} \end{aligned}$$

Two-component FQHE

- **Spin DOF in FQHE in GaAs**
  - Explicit Laughlin wavefunction

$$\Psi_{1/m}(\{z_i\}) = \prod_{j<k}^{N} \left(\frac{z_j - z_k}{\ell_B}\right)^m \exp\left\{-\frac{1}{4\ell_B^2}\sum_{\ell=1}^{N}|z_\ell|^2\right\}|\uparrow\uparrow\ldots\uparrow\rangle$$

$$\frac{B}{\Phi_0} = m\rho_e$$

  - Halperin's generalization of the Laughlin wavefunction

$$\Phi_{m,m',n}[Z] = \prod_{i<j\le N_\uparrow}(z_i - z_j)^m \prod_{k<l\le N_\downarrow}(z_{[k]} - z_{[l]})^{m'} \prod_{a=1}^{N_\uparrow}\prod_{b=1}^{N_\downarrow}(z_a - z_{[b]})^n \prod_{s=1}^{N}\exp\left\{-\frac{1}{4}|z_s|^2\right\}$$

$$[i] \equiv N_\uparrow + i$$

  - Flux-particle relation

$$\frac{B}{\Phi_0} = m\rho_\uparrow + n\rho_\downarrow \qquad \nu_\uparrow = \frac{\rho_\uparrow\Phi_0}{B} = \frac{m' - n}{mm' - n^2}$$

$$\frac{B}{\Phi_0} = m'\rho_\downarrow + n\rho_\uparrow \qquad \nu_\downarrow = \frac{\rho_\downarrow\Phi_0}{B} = \frac{m - n}{mm' - n^2}$$

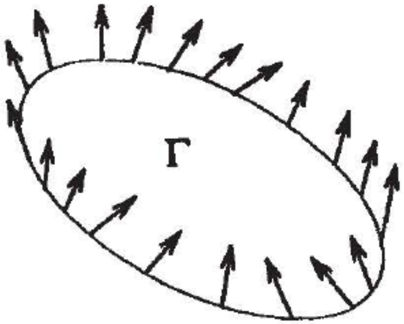
  - Case $\{m, m, m\}$ ⇒ Laughlin $\nu = 1/m$ state

$$\nu_\uparrow + \nu_\downarrow = \frac{1}{m} \qquad \nu_\uparrow - \nu_\downarrow = \frac{2S^z}{Nm}$$

Topological defects in FQHE

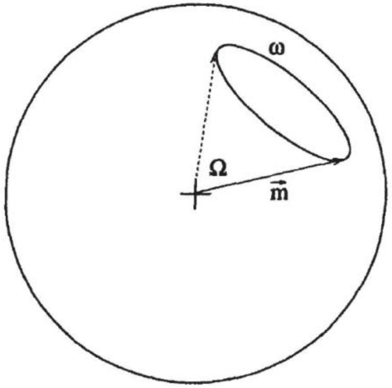
- **Skyrmions: closer look at Laughlin quasiparticle**
  - Real charge enclosed in region Γ

$$\Delta Q = -\frac{e\nu}{8\pi}\int_\Gamma dx\,dy\,\epsilon_{\mu\nu}\,\mathbf{m}\cdot\partial_\mu\mathbf{m}\times\partial_\nu\mathbf{m} \qquad |\mathbf{m}| = 1$$



  - Overall phase across Γ = Berry + Aharonov-Bohm

$$\Delta\Phi_{\text{total}} = \frac{\Omega}{4\pi}\Phi_0$$



  - Generalized Laughlin argument

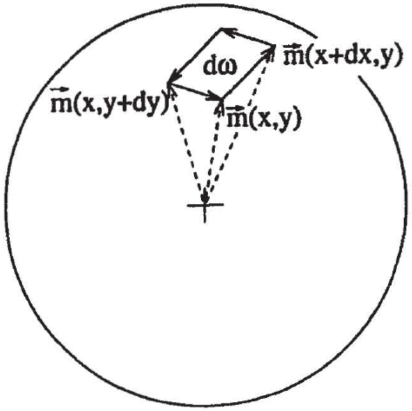
$$\oint_{\partial\Gamma}\mathbf{E}\cdot d\mathbf{r} = -\frac{d\Phi_{\text{total}}}{dt} \qquad \hat{\mathbf{z}}\cdot\mathbf{J}\times d\mathbf{r} = \sigma_{xy}\,\mathbf{E}\cdot d\mathbf{r}$$

$$\frac{dQ}{dt} = \sigma_{xy}\frac{d\Phi_{\text{total}}}{dt} \qquad \Rightarrow \qquad \Delta Q = -e\nu\frac{\Delta\Phi_{\text{total}}}{\Phi_0} = -e\nu\frac{\Omega}{4\pi}$$

  - Infinitesimal square loop

$$d\omega = [\mathbf{m}(x+dx,y)-\mathbf{m}(x,y)]\times[\mathbf{m}(x,y+dy)-\mathbf{m}(x,y)]\cdot\mathbf{m}(x,y)$$

$$\Omega = \int_\Gamma dx\,dy\,\frac{1}{2}\epsilon_{\mu\nu}\,\mathbf{m}\cdot\partial_\mu\mathbf{m}\times\partial_\nu\mathbf{m}$$



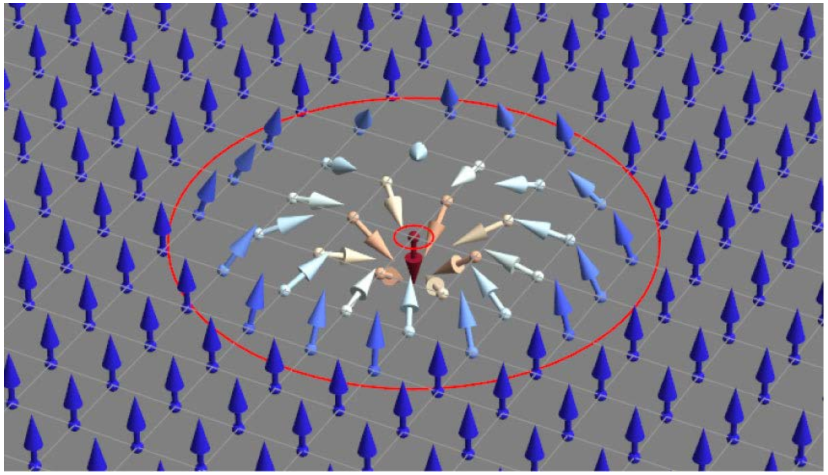
$$\boxed{\Delta Q = -\frac{e\nu}{8\pi}\int_\Gamma dx\,dy\,\epsilon_{\mu\nu}\,\mathbf{m}\cdot\partial_\mu\mathbf{m}\times\partial_\nu\mathbf{m}}$$

Topological defects in FQHE

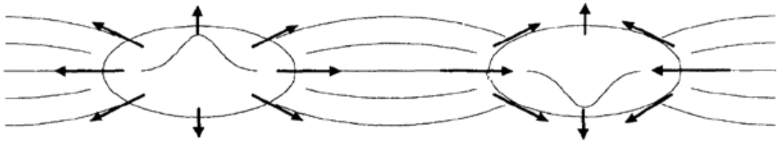
- **Skyrmions and merons**
  - Real charge enclosed in region Γ

$$\Delta Q = -\frac{e\nu}{8\pi}\int_\Gamma dx\,dy\,\epsilon_{\mu\nu}\,\mathbf{m}\cdot\partial_\mu\mathbf{m}\times\partial_\nu\mathbf{m}$$

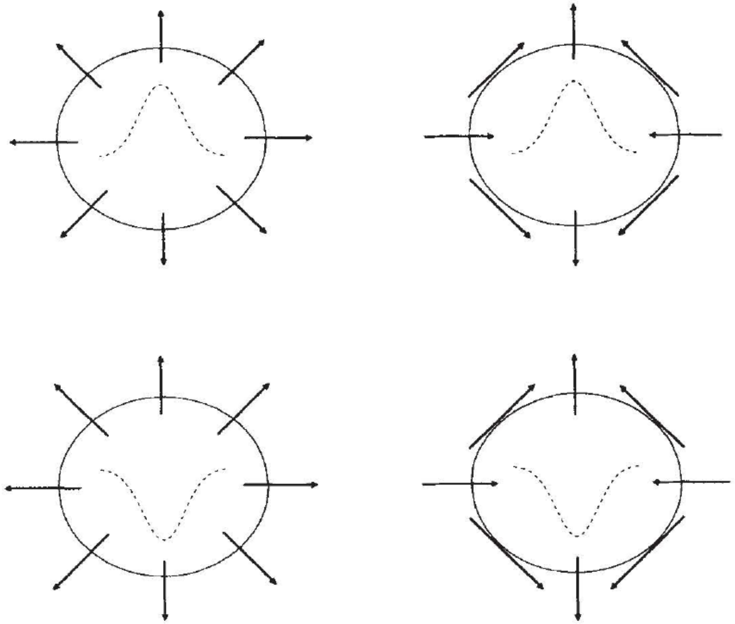
  - Skyrmion with unit winding traps *e*/*m* charge



  - Break SU(2) ⇒ meron = half skyrmion ⇒ charge = *e*/2*m*
  - Four types of merons
  - Skyrmions as **charged** meron bound states (gapped)



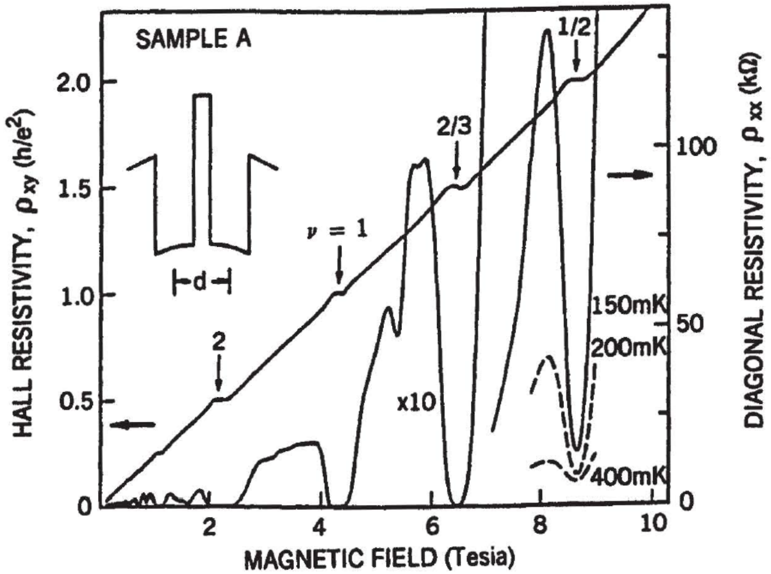
  - Neutral bound states ⇒ spin waves (gapless)



Bilayer FQHE

- **The ν = 1/2 "QHE"**
  - Layer DOF ⇒ pseudospin
  - ν = 1/2 = 1/4 + 1/4
  - Halperin wave function

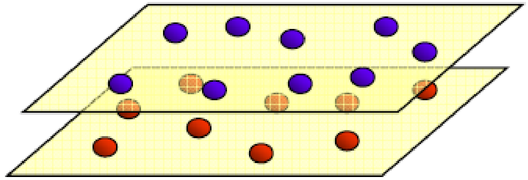
$$\Psi_{331} \sim \prod (z_i - z_j)^3 \prod (z_m - w_n) \prod (w_k - w_l)^3$$

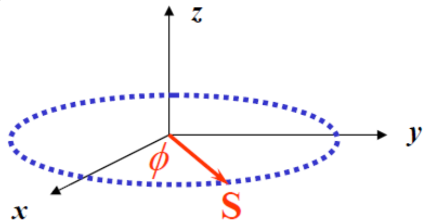


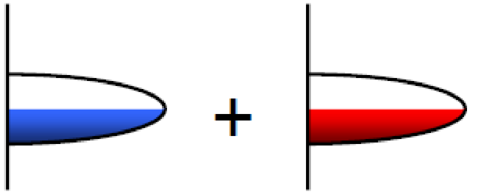
- **Superfluid at ν = 1 = 1/2 + 1/2**
  - Halperin wave function

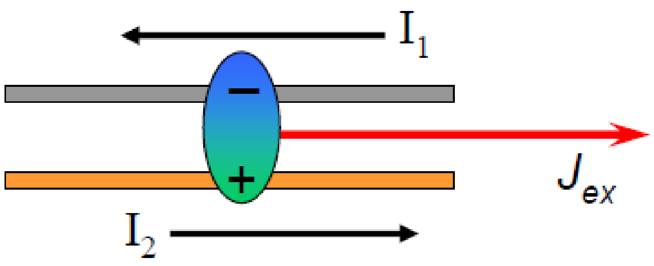
$$\Psi \sim \prod (z_i - z_j)(w_k - w_l)(z_m - w_n)$$

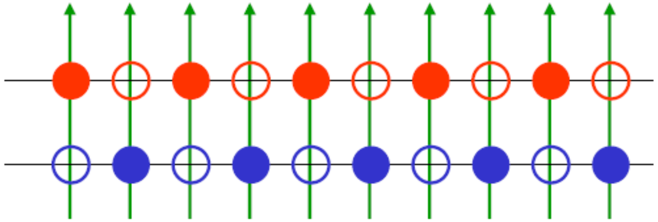
$$|\Psi\rangle = \prod_{\mathbf{k}} |\mathbf{k}\rangle \otimes \frac{1}{\sqrt{2}} \left[|\uparrow\rangle + e^{i\phi} |\downarrow\rangle \right]$$







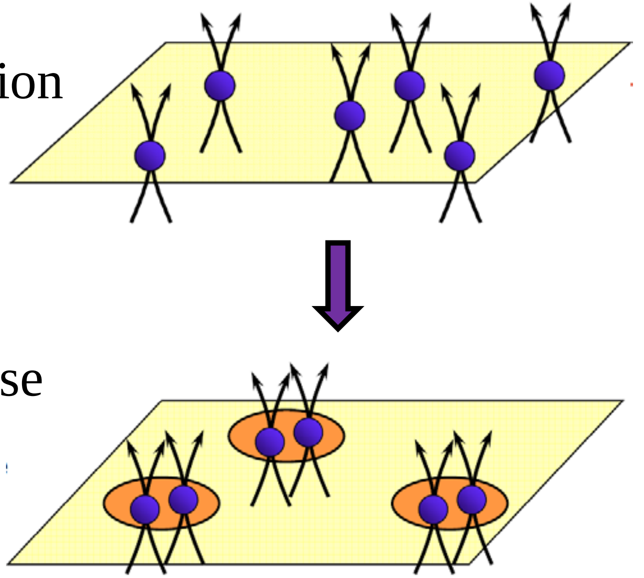




Moore-Read state

- **The Pfaffian wave function**
  - Recall Halperin-Lee-Read (HLR) wave function

$$\Psi_{1/2} = \begin{vmatrix} e^{i\mathbf{k}_1\cdot\mathbf{r}_1} & e^{i\mathbf{k}_1\cdot\mathbf{r}_2} & \cdots \\ e^{i\mathbf{k}_2\cdot\mathbf{r}_1} & \ddots & \\ \vdots & & \end{vmatrix} \prod_{i<j}(z_i - z_j)^2 \prod_i e^{-\frac{1}{4}|z_i|^2}$$



  - Wave function for the Moore-Read (MR) phase

$$\Psi^{\mathrm{Pf}}_{1/2} = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^2 \prod_i e^{-\frac{1}{4}|z_i|^2}$$

$$\mathrm{Pf}(M_{ij}) = A\left[M_{12}M_{34}\ldots M_{N-1,N}\right]$$

$$\Psi^{\mathrm{Pf}}_{1/2}(z_1, z_2, z_3, z_4) = \left[\frac{1}{(z_1 - z_2)(z_3 - z_4)} - \frac{1}{(z_1 - z_3)(z_2 - z_4)} + \frac{1}{(z_1 - z_4)(z_2 - z_3)}\right]$$

$$\times (z_1 - z_2)^2(z_1 - z_3)^2(z_1 - z_4)^2(z_2 - z_3)^2(z_2 - z_4)^2(z_3 - z_4)^2$$

$$\times \exp\left[-\frac{1}{4}|z_1|^2 - \frac{1}{4}|z_2|^2 - \frac{1}{4}|z_3|^2 - \frac{1}{4}|z_4|^2\right]$$

  - BCS wave function ansatz

$$|\Psi_{\mathrm{BCS}}\rangle = \prod_{\mathbf{k}}{}' \prod_{\alpha}\left(u_{\mathbf{k}\alpha\alpha} + \sum_{\beta} v_{\mathbf{k}\alpha\beta} c^{\dagger}_{\mathbf{k}\alpha} c^{\dagger}_{-\mathbf{k}\beta}\right)|0\rangle$$

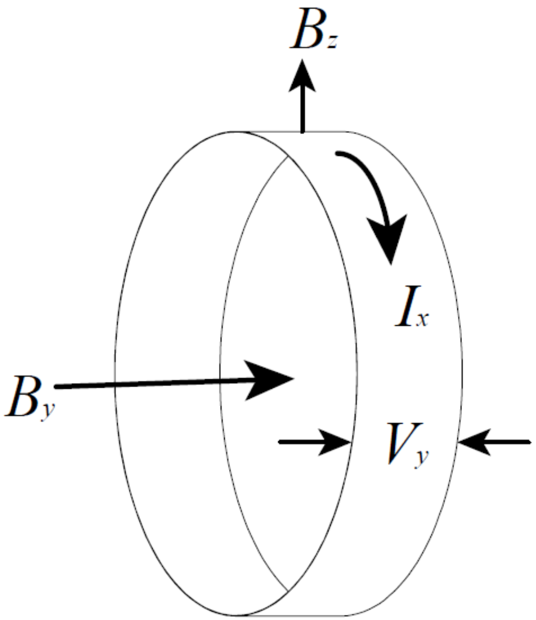
$$\Psi_{\mathrm{BCS}}(\{\mathbf{r}_i\}) = A\left[\phi(\mathbf{r}_1, \mathbf{r}_2)\phi(\mathbf{r}_3, \mathbf{r}_4)\ldots\phi(\mathbf{r}_{N-1}, \mathbf{r}_N)\right]$$

	$B = 0$	$B \neq 0$
Metal	Fermi liquid	HLR phase
Superconductor	BCS	MR phase

Back to gauge arguments

- **QHE vs. Superconductivity**
  - QHE: Laughlin gauge argument

$$\Psi = \exp\left[-\mathrm{i}\frac{e}{\hbar c}\int^{x} \mathbf{A}_{\mathrm{t}} \cdot d\mathbf{l}\right]\Psi'$$



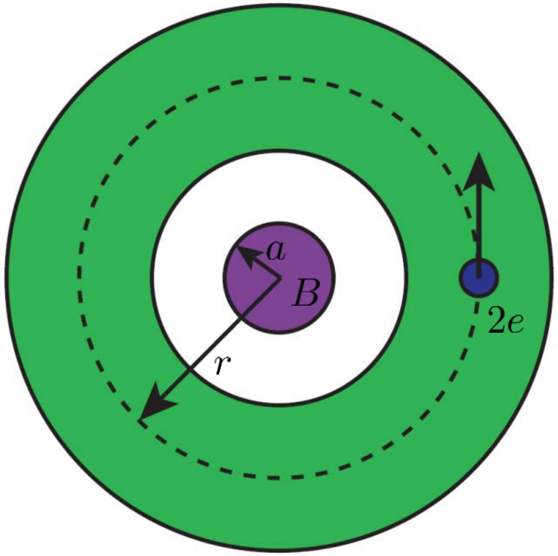
  - Bottom-line: **no** excitations smaller than ϕ_0

$$\nu = \frac{\nu^*}{2p\nu^* + 1} \qquad Q = \frac{-e}{2p\nu^* + 1}$$

  - Superconductor: complex order parameter $\Delta = \Delta_0 \mathrm{e}^{\mathrm{i}\theta}$
  - With cooper pair charge $q = 2e$ and single-valued Δ

$$\tilde{\phi}_0 = \frac{h}{2e} \approx 2 \times 10^{-11}\ \mathrm{T} \cdot \mathrm{cm}^2 = \frac{\phi_0}{2}$$

  - Moore-Read state: only 2 quasihole excitations possible
  - Each quasihole carries “e/4” quasiparticle



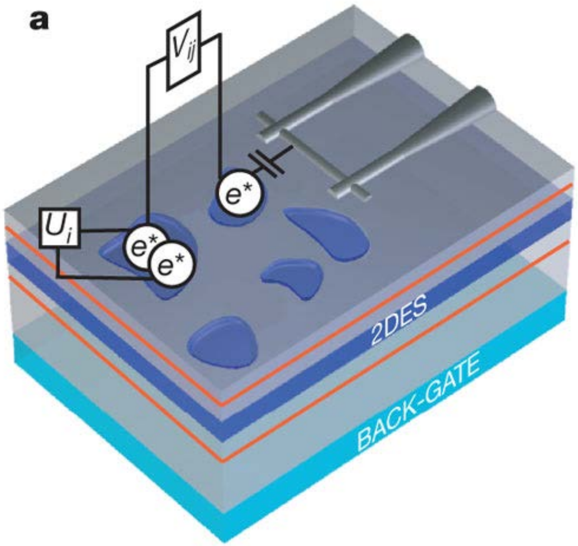
Detecting Moore-Read quasiparticle

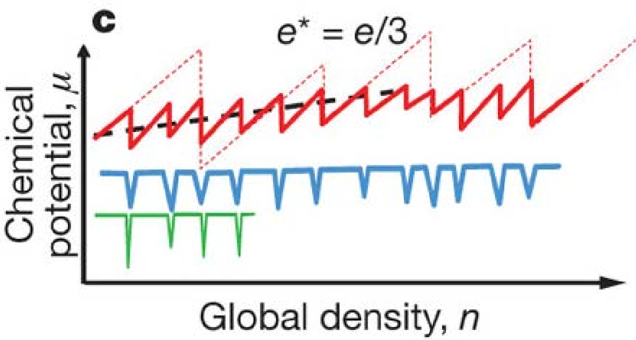
- **Local compressibility measurements**
  - Ratio of charge quasiparticles

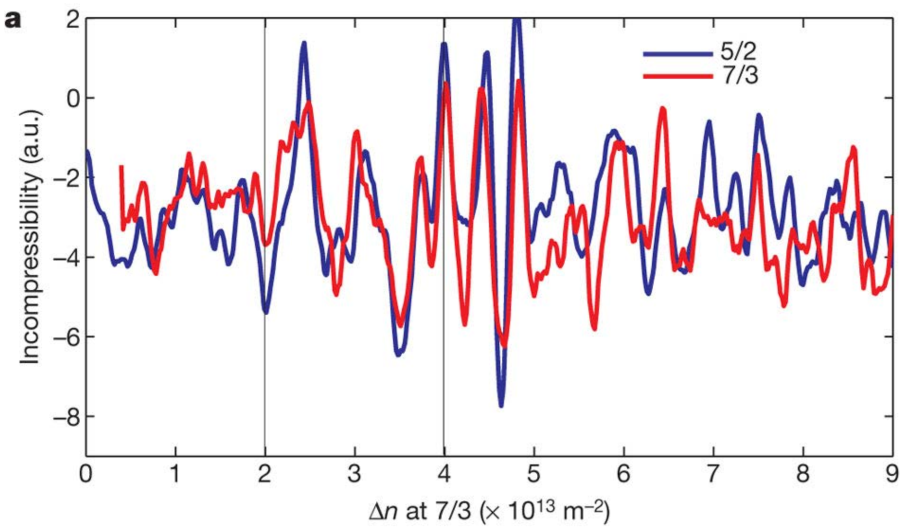
$$\beta = \frac{e^*_{7/3}}{e^*_{5/2}} = \frac{e/3}{e/4} = 1.33$$

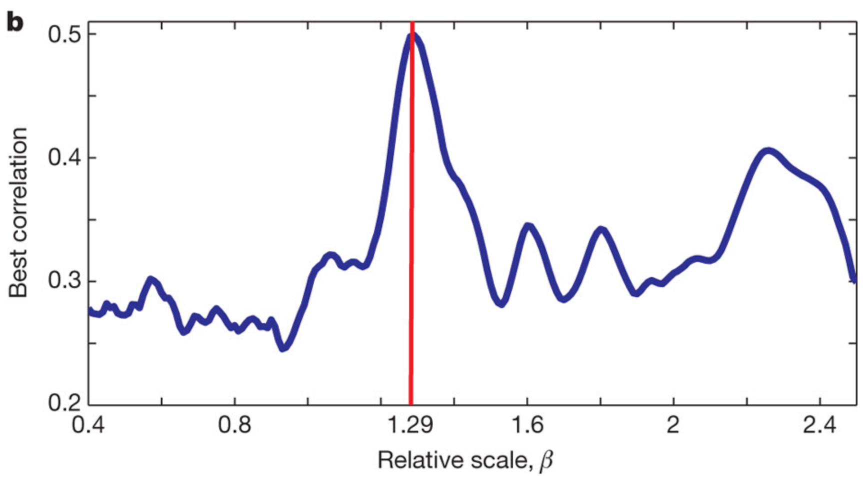
  - Measure correlation function

$$\frac{\langle C_{5/2}(i) C_{7/3}(i)\rangle}{\sqrt{\langle C^2_{5/2}(i)\rangle \langle C^2_{7/3}(i)\rangle}}$$



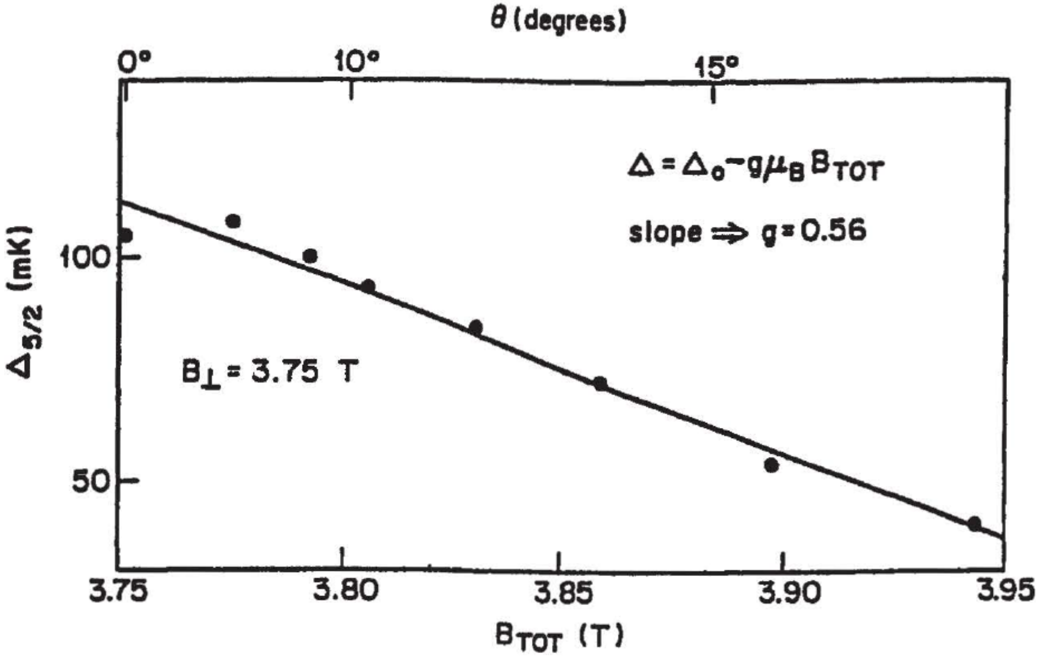


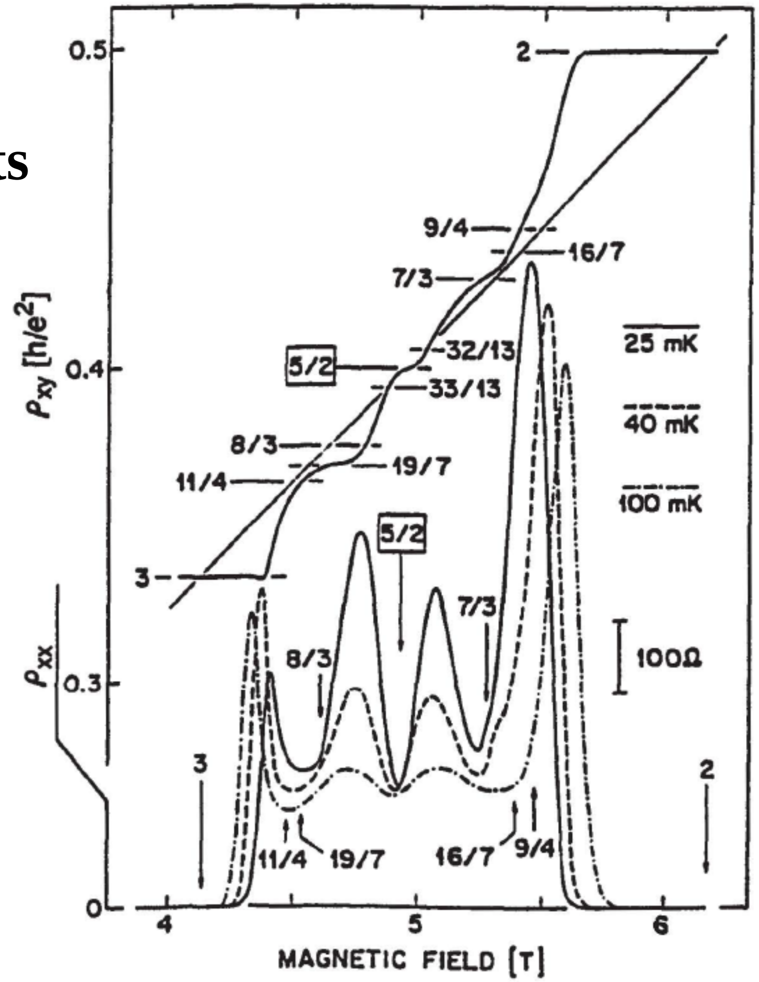


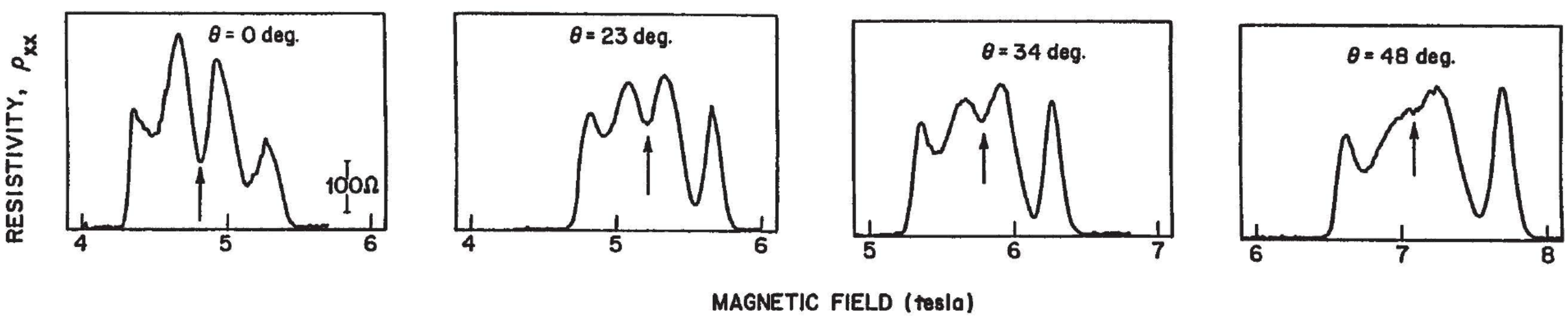


ν = 5/2: polarized or unpolarized?

- **Instability in tilted field measurements**

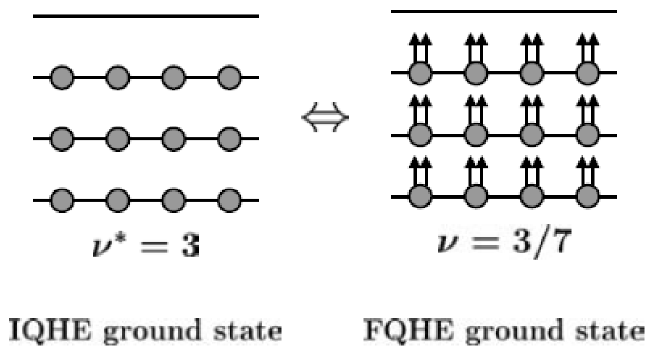


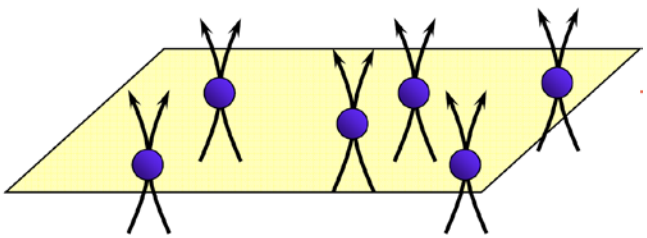




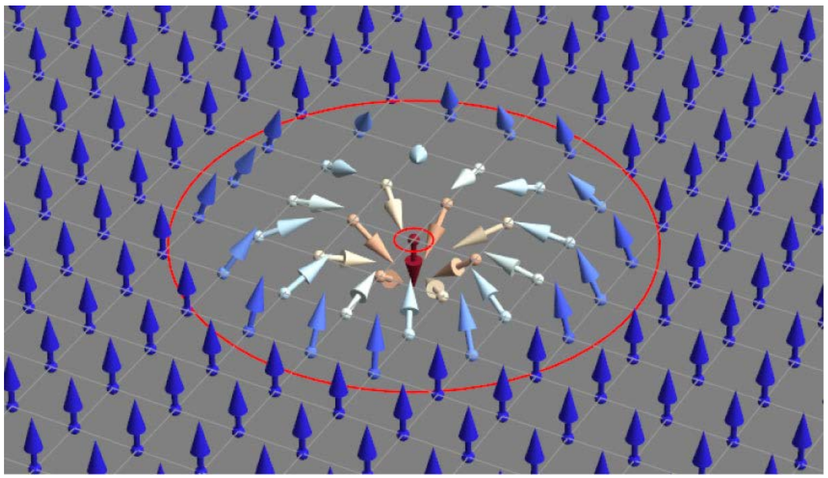
Summary and Conclusions

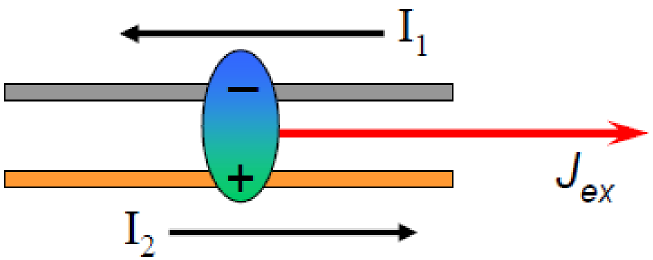
- **Part 1: Composite fermions**
  - Convenient FQHE to IQHE mapping
  - Halperin-Lee-Read (HLR): metallic state at **single layer** $\nu = 1/2$





- **Part 2: Multicomponent FQHE**
  - Quantum Hall Ferromagnetism
  - Topological spin excitations
  - Excitonic superfluidity in bilayer system





Thanks for listening

Questions?