Introduction to even-denominator FQHE: composite fermions

Tejas Deshpande

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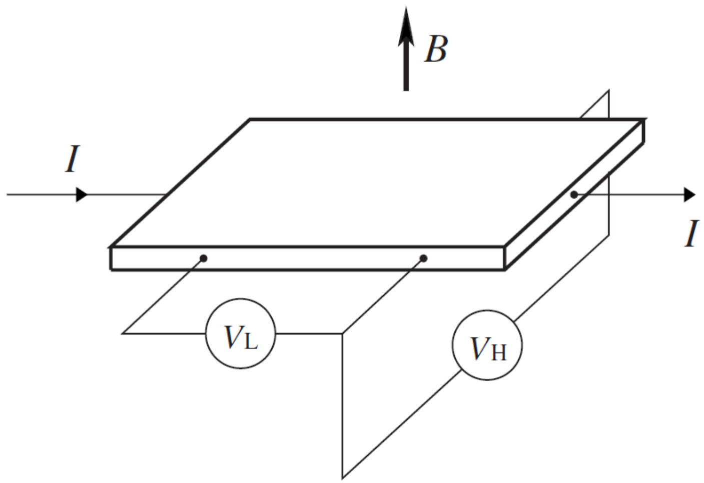
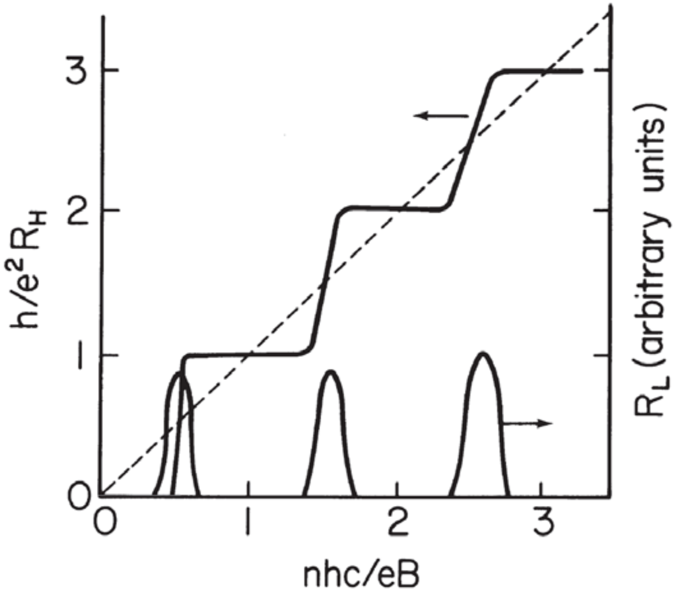
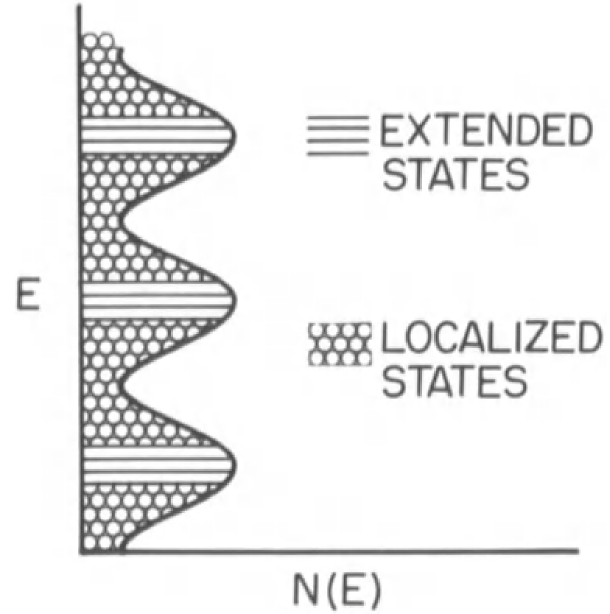
Integer Quantum Hall Effect

- **Phenomenology**
  - von Klitzing sees IQHE in silicon MOSFET in 1980
  - Landau level Filling fraction or Hall conductivity:

$$\nu = \left(\frac{e^2}{h}\right)^{-1} \frac{I}{V_{\mathrm{H}}} = \frac{N_\phi}{N_e}$$







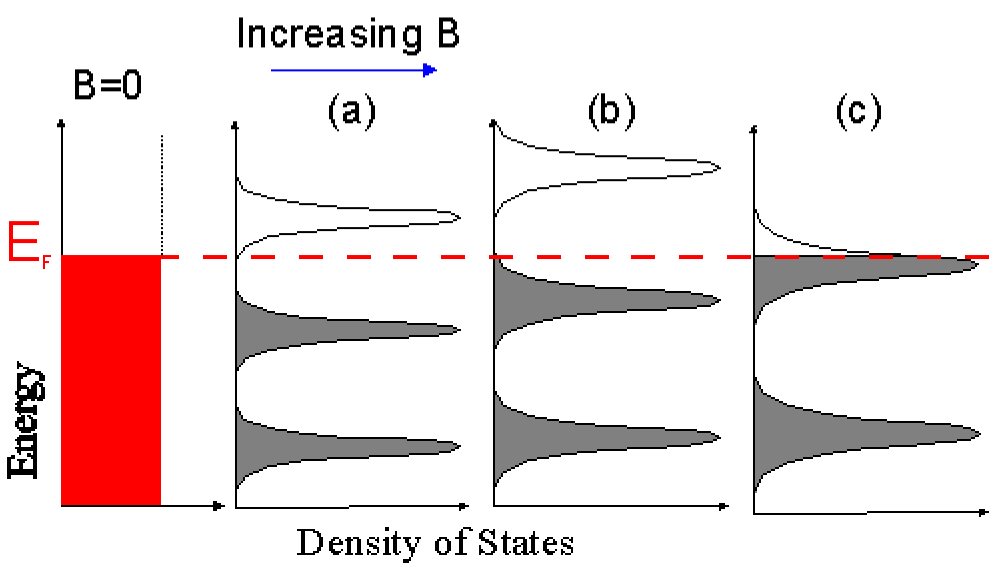
  - Landau level degeneracy = **total** number of flux quanta (N_ϕ)

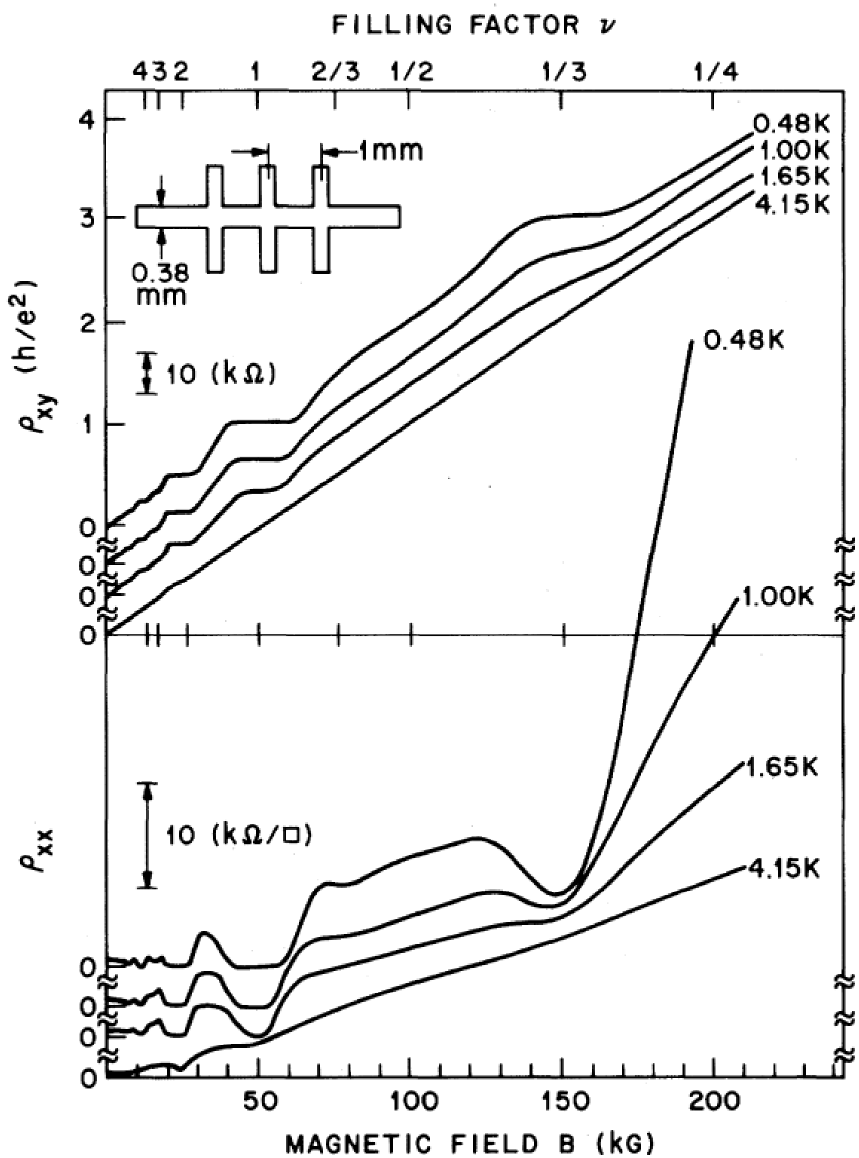
Fractional Quantum Hall Effect

- **Phenomenology**
  - Tsui & Stormer see $\nu = 1/3$ in GaAs quantum well in 1982
  - Landau level Filling fraction or Hall conductivity:

$$\nu = \left(\frac{e^2}{h}\right)^{-1} \frac{I}{V_{\mathrm{H}}} = \frac{N_\phi}{N_e}$$

  - Partially filled **non-interacting** Landau level ($n + p/q \equiv p/q$)





Aharonov-Bohm Effect

- **Quantum effect that lacks gauge invariance**
  - Phase picked up by a quantum particle of charge q:

$$\varphi = \frac{q}{\hbar}\int_C \mathbf{A}(\mathbf{x})\cdot d\mathbf{x}$$

  - Gauge choice and parameterization

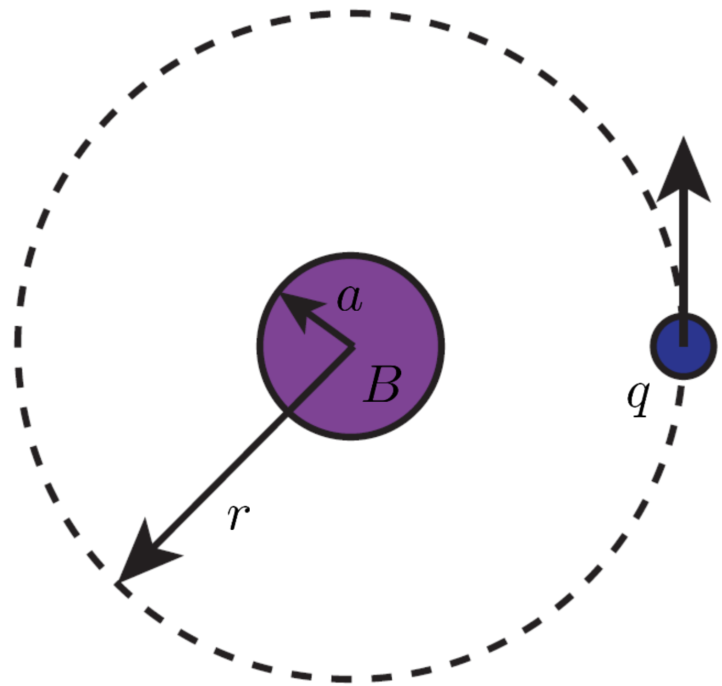
$$A_\theta(r) = \frac{Ba^2}{2r} = \frac{B(\pi a^2)}{2\pi r} = \frac{\Phi}{2\pi r}$$

  - Phase picked up for arc subtending angle θ in the circle

$$\varphi = \frac{q}{\hbar}\left[\frac{\Phi}{2\pi r}(r\theta)\right] = \theta\frac{N\phi_0 q}{h}$$

  - Quantum of flux is defined as: $\phi_0 = \frac{h}{e} \approx 4\times 10^{-11}\ \mathrm{T}\cdot\mathrm{cm}^2 \Rightarrow \quad \varphi = \left(\frac{q}{e}\right)N\theta$
  - For $q = e$ and $\theta = \pi$ we have:

$$\mathrm{e}^{\mathrm{i}\varphi} = \begin{cases} +1 & N \text{ even} \\ -1 & N \text{ odd} \end{cases}$$



"Gauge invariance" in many disguises

- **Laughlin's gauge argument**
  - Quantization of Hall resistance from thought experiment

$$0 \leq x \leq L \qquad 0 \leq y \leq W$$

$$\phi_{\mathrm{t}} = \frac{B_y L^2}{4\pi} \equiv N\phi_0 \qquad \phi = B_z L W \equiv N_\phi \phi_0$$

  - Gauge choices

$$\mathbf{A}_{\mathrm{t}} = \frac{N\phi_0}{L}\hat{\mathbf{x}} \qquad \mathbf{A} = -B_z y\, \hat{\mathbf{x}}$$

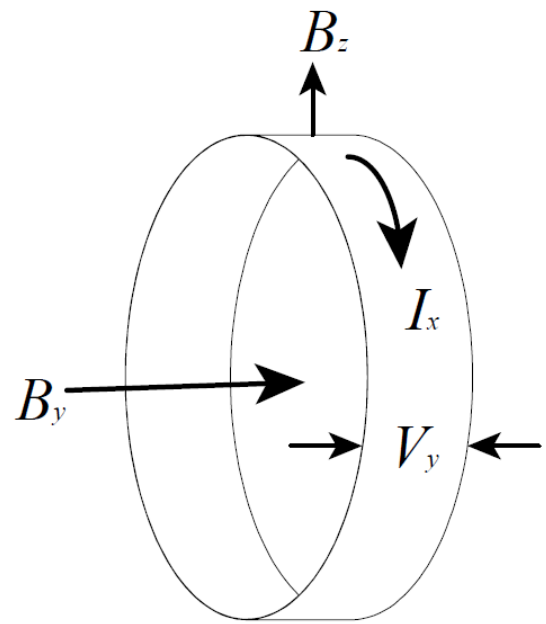
  - Hamiltonian

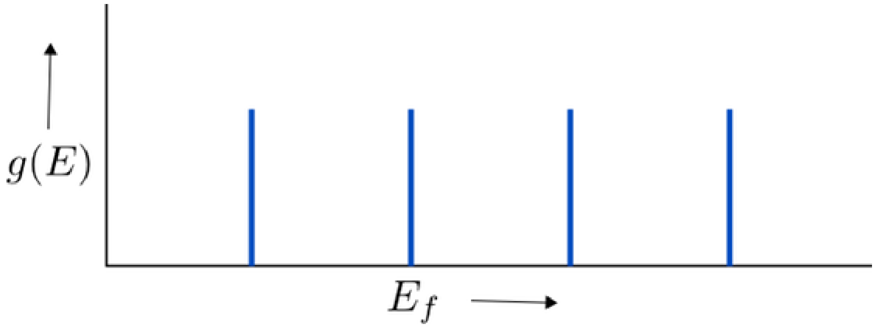
$$H = \frac{1}{2m_{\mathrm{b}}}\left(\hbar\mathbf{k} + \frac{e}{c}\mathbf{A} + \frac{e}{c}\mathbf{A}_{\mathrm{t}}\right)^2$$

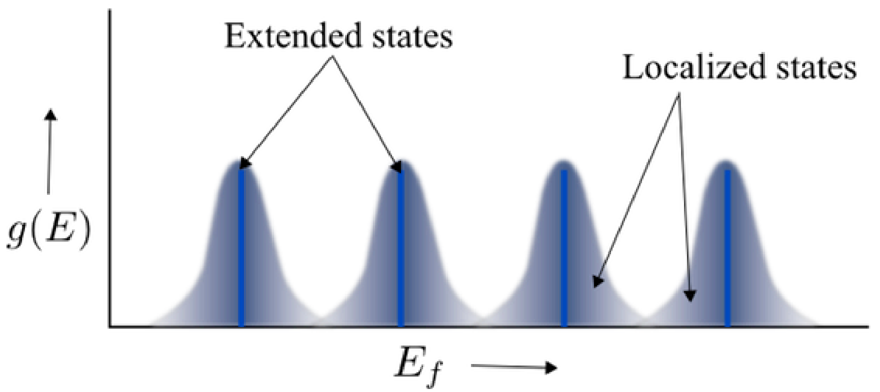
$$\Psi = \exp\left[-\mathrm{i}\frac{e}{\hbar c}\int^x \mathbf{A}_{\mathrm{t}}\cdot d\mathbf{l}\right]\Psi'$$

$$\Psi'(x,y) = \Phi(y)\mathrm{e}^{\mathrm{i}k_x x}$$

  - Periodicity in *x*: $k_x = \frac{2\pi}{L}(j - N)$







"Gauge invariance" in many disguises

- **Superconductivity**
  - Also a macroscopic quantum effect like QHE
  - Complex order parameter $\Delta = \Delta_0 e^{i\theta}$
  - Flux quantization (Aharonov-Bohm effect)

$$\varphi = \frac{q}{\hbar}\int_C \mathbf{A}(\mathbf{x}) \cdot d\mathbf{x}$$

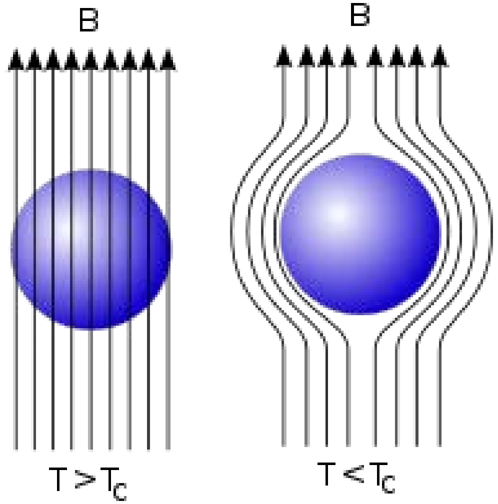
$$A_\theta(r) = \frac{Ba^2}{2r} = \frac{B(\pi a^2)}{2\pi r} = \frac{\tilde{\phi}_0}{2\pi r}$$

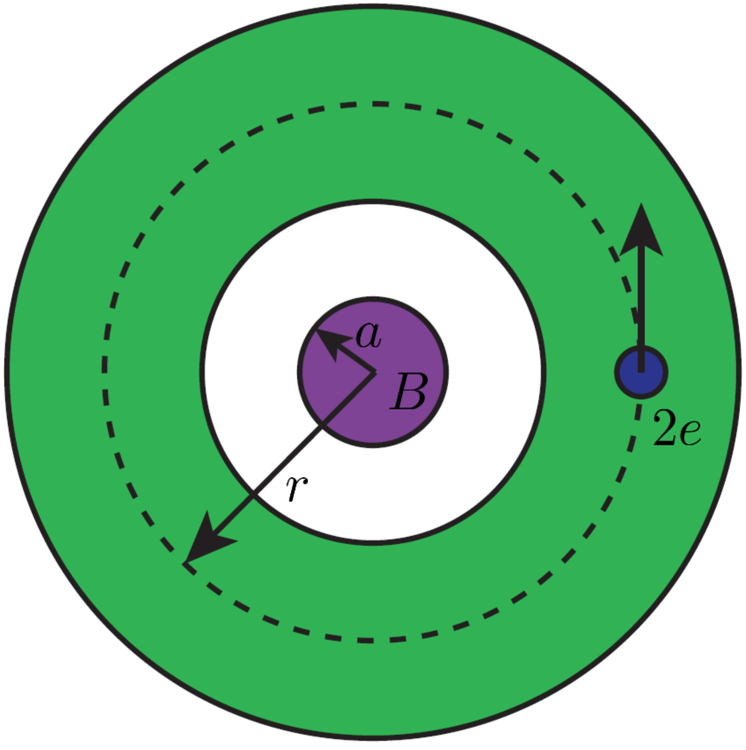
$$\varphi = \frac{q}{\hbar}\left[\frac{\tilde{\phi}_0}{2\pi r}(r\theta)\right] = \theta\frac{\tilde{\phi}_0 q}{h}$$

  - With cooper pair charge $q = 2e$ and single-valued Δ

$$\tilde{\phi}_0 = \frac{h}{2e} \approx 2\times 10^{-11}\ \mathrm{T\cdot cm^2} = \frac{\phi_0}{2}$$

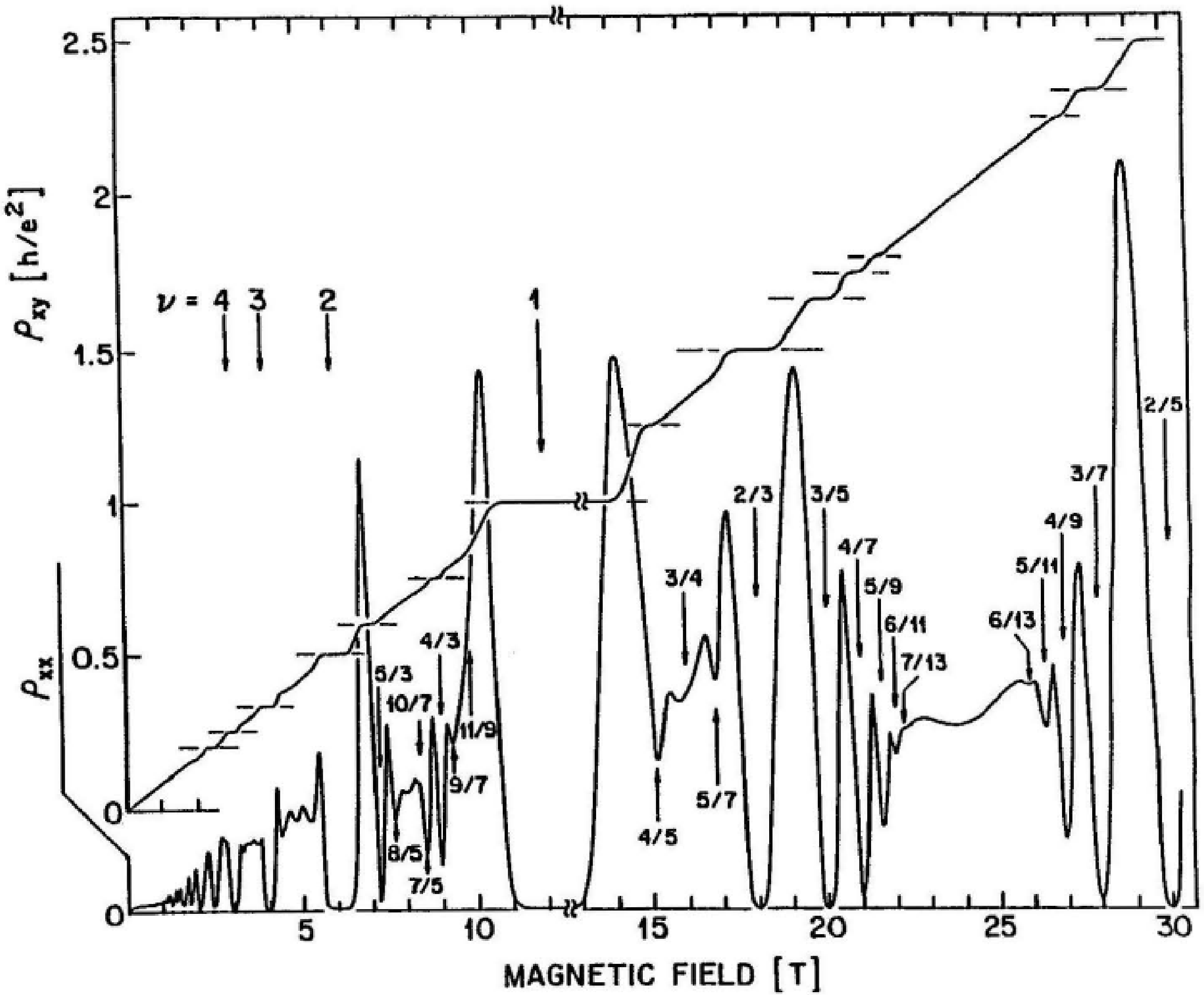
  - Fundamental principle behind SQUIDs (Josephson effect)





FQHE: Review of Laughlin states

- **Experimental Background**
  - Wide range of fractions discovered after the 1982 discovery



FQHE: Review of Laughlin states

- **Theoretical Background**

  - Challenge: solve Schrodinger's equation

$$H\Psi = E\Psi$$

$$H = \sum_j \frac{1}{2m_\mathrm{b}} \left[\frac{\hbar}{i} \boldsymbol{\nabla}_j + \frac{e}{c} \mathbf{A}(\mathbf{r}_j) \right]^2 + \frac{e^2}{\epsilon} \sum_{j<k} \frac{1}{|\mathbf{r}_j - \mathbf{r}_k|} + \sum_j U(\mathbf{r}_j) + g\mu \mathbf{B} \cdot \mathbf{S}$$

  - Important energy/length scales

$$\hbar\omega_\mathrm{c} = \hbar \frac{eB}{m_\mathrm{b} c} \approx 20\, B[\mathrm{T}]\ \mathrm{K} \qquad V_\mathrm{C} \equiv \frac{e^2}{\epsilon \ell_\mathrm{B}} \approx 50 \sqrt{B[\mathrm{T}]}\ \mathrm{K}$$

$$E_\mathrm{Z} = 2g\mu_\mathrm{B} \mathbf{B} \cdot \mathbf{S} = \frac{g}{2} \left(\frac{m_\mathrm{b}}{m_\mathrm{e}} \right) \hbar\omega_\mathrm{c} \approx 0.3\, B[\mathrm{T}]\ \mathrm{K}$$

$$\ell_\mathrm{B} = \left(\frac{\hbar c}{eB} \right)^{1/2} \approx \frac{25}{\sqrt{B[\mathrm{T}]}}\ \mathrm{nm}$$

Units

$$\ell_\mathrm{B} = 1$$

$$\frac{e^2}{\epsilon \ell_\mathrm{B}} = 1$$

  - Laughlin writes **many-body** wavefunction *ansatz* for a specific set of FQHE states with ν = 1/*m*

$$\Psi_{1/m}(\{z_i\}) = \prod_{j<k}^{N} (z_j - z_k)^m \exp\left\{ -\frac{1}{4} \sum_{\ell=1}^{N} |z_\ell|^2 \right\} \qquad z_j = x_j + \mathrm{i} y_j$$

FQHE: Review of Laughlin states

- **Laughlin wavefunction**
  - Laughlin wavefunction $\nu = 1/m$ FQHE states (with units)

$$\Psi_{1/m}(\{z_i\}) = \prod_{j<k}^{N} \left(\frac{z_j - z_k}{\ell_\mathrm{B}} \right)^m \exp\left\{ -\frac{1}{4\ell_\mathrm{B}^2} \sum_{\ell=1}^{N} |z_\ell|^2 \right\}$$

  - What does it mean?

1. It vanishes as *any* two electrons approach each other due to:

$$\prod_{j<k}^{N} (z_j - z_k)^m \quad \Rightarrow \quad \text{repulsion between electrons}$$

2. Larger m ⇒ electrons farther apart ⇒ larger angular momentum

$$m = \frac{N_\phi}{N_e} \quad \Rightarrow \quad \text{larger } m \Rightarrow \text{lower electron density}$$

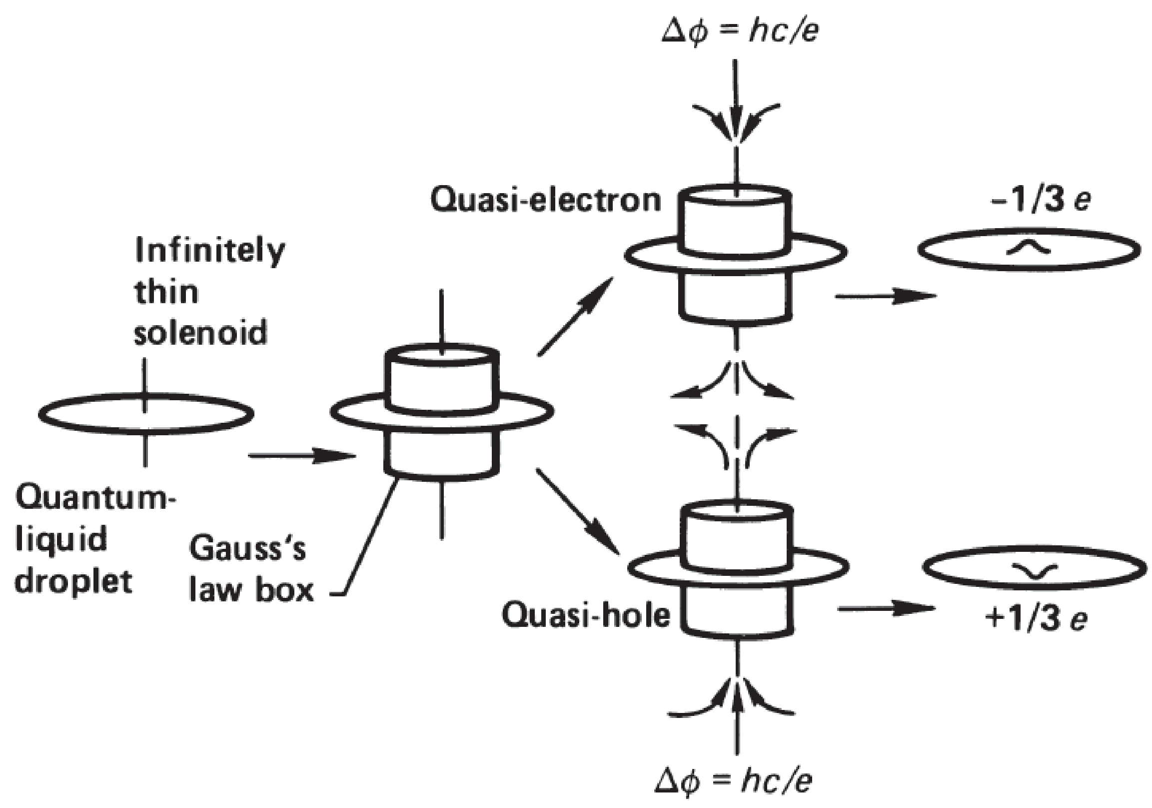
3. Electrons within ℓ_B feel “attraction” to the origin; analogous to classical 2D plasma ⇒ Boltzmann probability distribution

$$|\Psi_m(\{z_i\})|^2 = \exp\{-\beta\phi(\{z_i\})\}$$

$$\phi(\{z_i\}) = -2m^2 \sum_{j<k} \ln|z_j - z_k| + \frac{m}{2} \sum_{\ell=1}^{N} |z_\ell|^2$$

FQHE: Review of Laughlin states

- **Laughlin quasiparticle**
  - Laughlin wave function describes ground state of **charge *e* electrons**
  - Where do charge ***e*/*m*** **anyons** come from?
  - Excitations create anyons. What causes excitations?

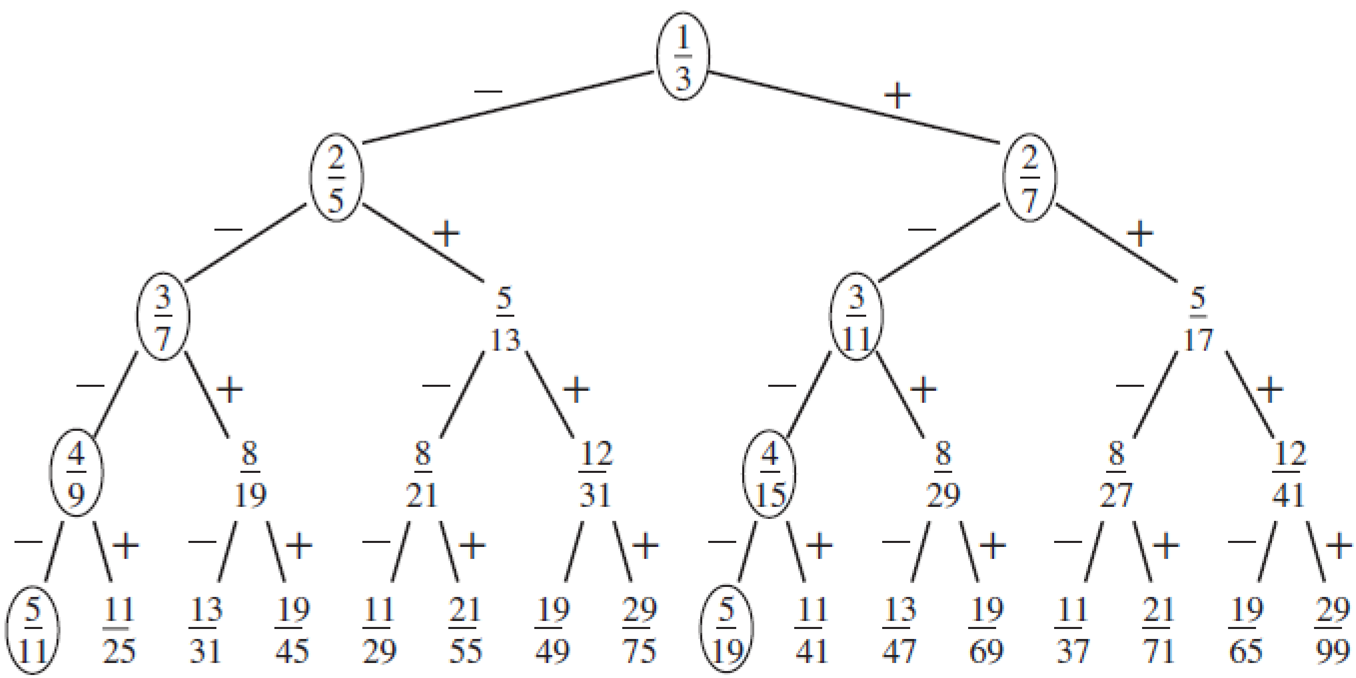


FQHE: Haldane's hierarchical structure

- **"Daughter" states**
  - Haldane proposed hierarchical construction for other fractions

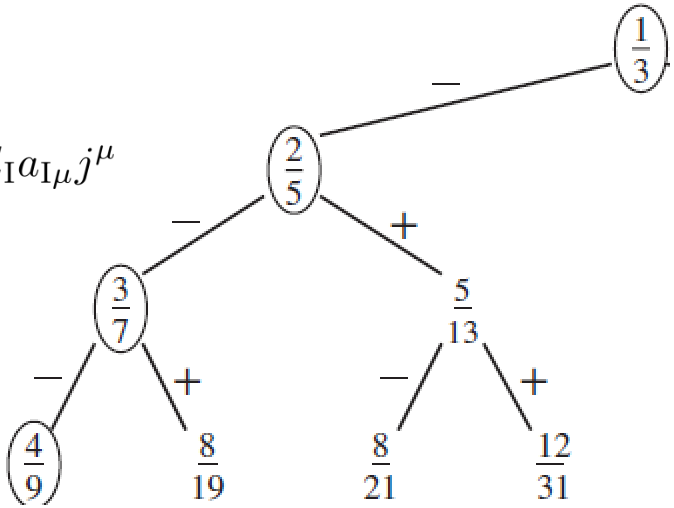
$$f = \cfrac{1}{m \pm \cfrac{1}{p_2 \pm \cfrac{1}{p_3 \pm \cfrac{1}{p_4 \pm \cfrac{1}{p_5}}}}}$$

  - FQHE of Laughlin quasiparticles of the 1/*m* state



FQHE: Review of the K-matrix

- **Example of ν = 2/5 and 3/7: “Shut up and calculate!”**



  - Chern-Simons effective theory

$$\mathcal{L} = -\frac{1}{4\pi}\varepsilon^{\mu\nu\lambda}K_{\mathrm{IJ}}a_{\mathrm{I}\mu}\partial_\nu a_{\mathrm{J}\lambda} + \frac{e}{2\pi}q_{\mathrm{I}}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_{\mathrm{I}\lambda} + l_{\mathrm{I}}a_{\mathrm{I}\mu}j^\mu$$

  - Formulas for ν and quasiparticle charge (Q)

$$\nu = \mathbf{q}^T K^{-1}\mathbf{q} \qquad Q = |e|\mathbf{q}^T K^{-1}\mathbf{l}$$

  - ν = 2/5 from Laughlin state ν = 1/3

$$\mathbf{q} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad K = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix} \quad \mathbf{l} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\nu = \begin{pmatrix} 1 & 0 \end{pmatrix}\left\{\frac{1}{5}\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}\right\}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{2}{5} \qquad Q = |e|\begin{pmatrix} 1 & 0 \end{pmatrix}\left\{\frac{1}{5}\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}\right\}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\frac{|e|}{5}$$

  - ν = 3/7 from daughter state ν = 2/5

$$\mathbf{q} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad K = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \quad \mathbf{l} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

  - K matrix dimension equal to the hierarchy level

Composite fermion "trick" (or theory)?

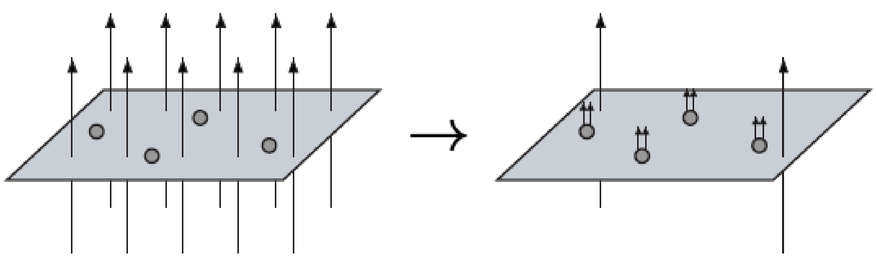
- **Mapping to IQHE**
  - Composite particle by "attaching" even (2*p*) flux quanta (*h*/*e*)

$$B = B^* + 2p\rho\phi_0$$

  - Real and effective filling factors

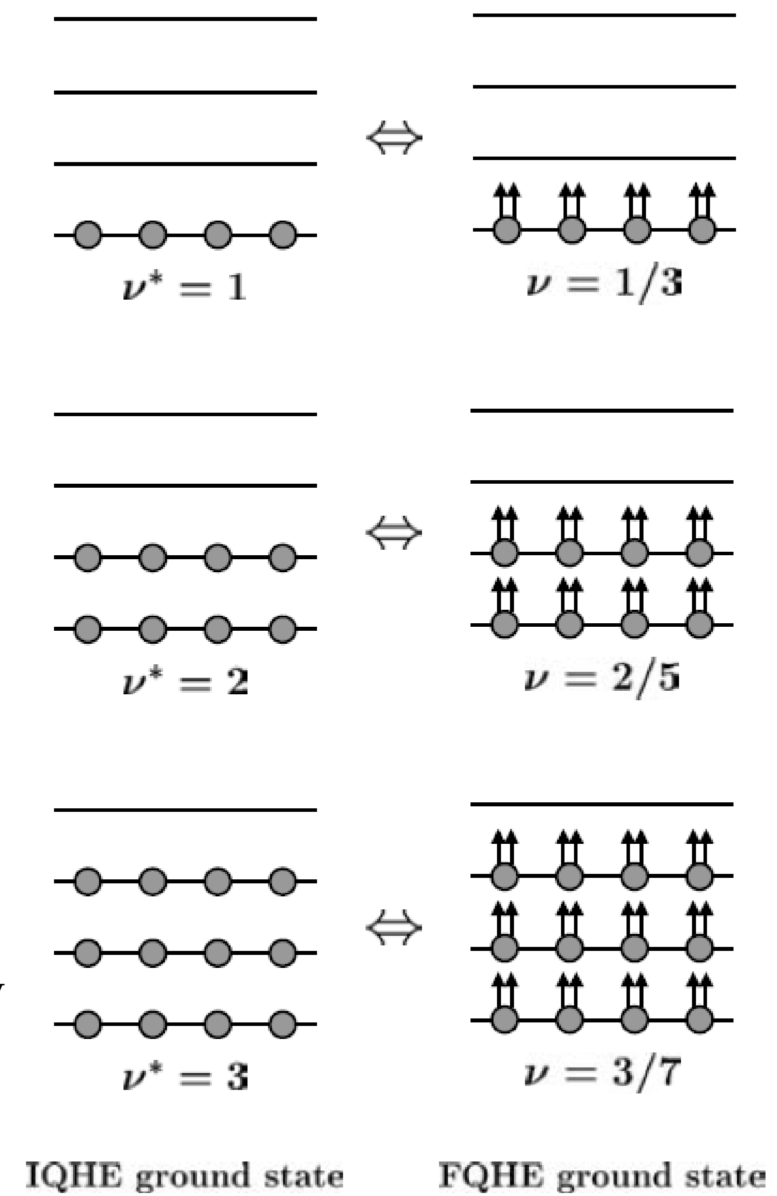
$$\nu = \frac{\rho\phi_0}{B} \qquad \nu^* = \frac{\rho\phi_0}{B^*} \qquad \nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

  - Physical picture of flux attachment: real electron + 2*p* fluxes ⇒ "composite fermion" (CF)



  - CF Landau level (Λ level) degeneracy = total **effective** flux quanta

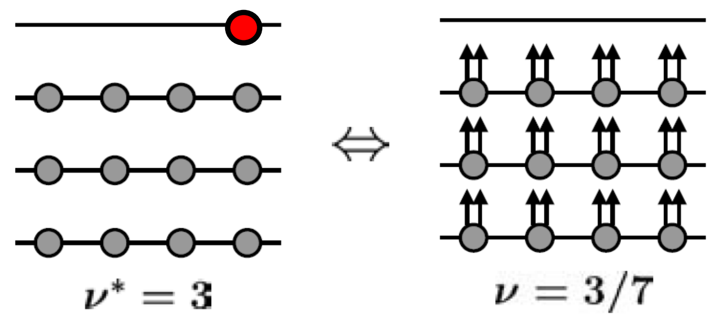
$$N_\phi^{\text{eff}} = N_\phi - 2pN_e$$



Composite fermion "trick" (or theory)?

- **What about Laughlin quasiparticles?**
  - CFs carry charge -*e* and spin 1/2
  - Consider ground state at *arbitrary* ν (ν*)

$$\nu = \frac{\nu^*}{2p\nu^* + 1}$$



  - Degeneracy of each Λ level:

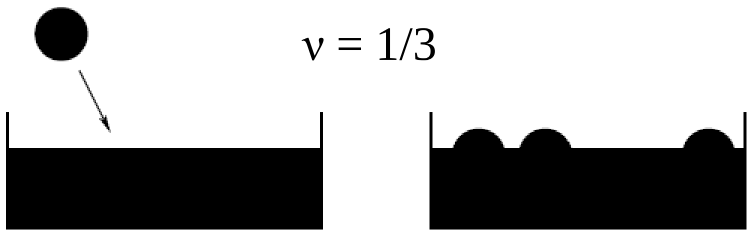
$$N_\phi^{\text{eff}} = N_\phi - 2pN_e$$

  - Add a **real** electron to (ν^* + 1)th Λ level (*local* excitation)
  - Modified degeneracy of each Λ level: $\tilde{N}_\phi^{\text{eff}} = N_\phi - 2p(N_e + 1)$

$$= N_\phi^{\text{eff}} - 2p$$

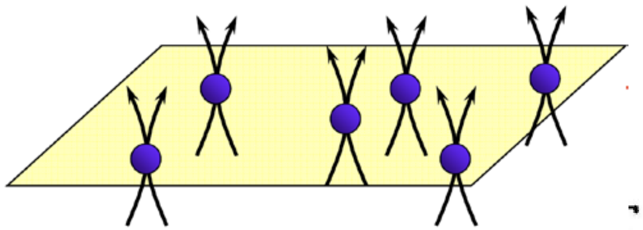
  - Each Λ level contributes to 2*p* CF-quasiparticle excitations
  - Total ($2p\nu^*$ + 1) CF-quasiparticles = external electron
  - Charge on each CF quasiparticle

$$Q = \frac{-e}{2p\nu^* + 1}$$

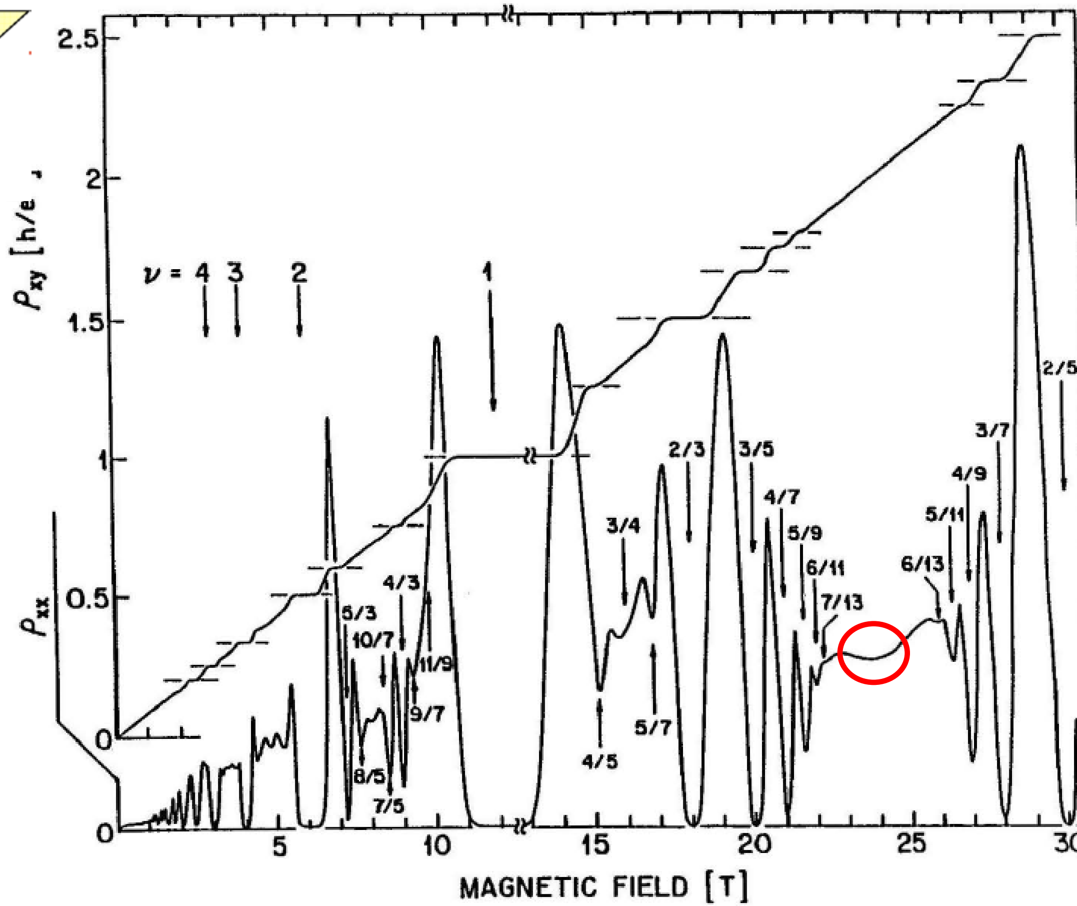


Halperin-Lee-Read (HLR) phase

- **Theory of the half-filled "Landau level"?**
  - Does the composite fermion picture have *any* predictive power?
  - HLR make hypothesis if Fermi liquid state at $\nu = 1/2$ in 1993



- Dips at other even denominator fractions
- No QHE at even denominators!
- No plateaus at even denominators



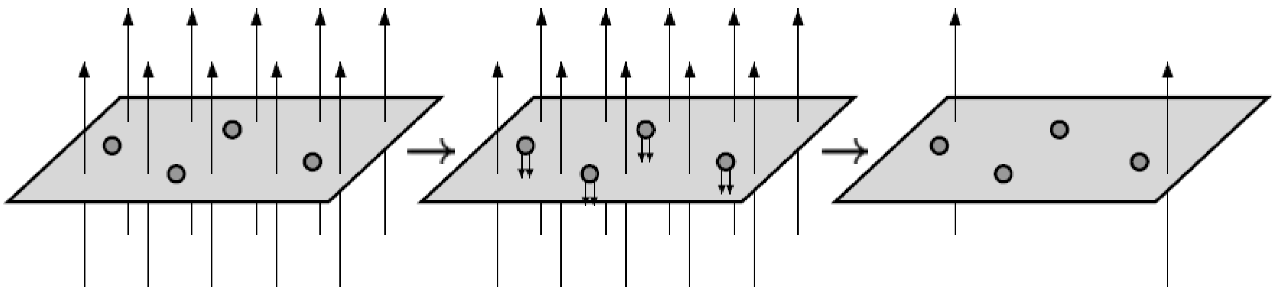
HLR phase: Chern-Simons Theory

- **Mathematical flux attachment**
  - Simplest Hamiltonian of FQHE

$$H = \frac{1}{2m^*}\int d^2\mathbf{r}\ \psi_{\mathrm{e}}^{\dagger}(-\mathrm{i}\boldsymbol{\nabla} + e\mathbf{A})^2\psi_{\mathrm{e}} + V$$

  - Attach flux via gauge transformation

$$\psi^{\dagger}(\mathbf{r}) \equiv \psi_{\mathrm{e}}^{\dagger}\exp\left(2\pi\mathrm{i}\tilde{\phi}\int d^2\mathbf{r}'\ \arg(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')\right)$$



  - Transformed Hamiltonian

$$H = \frac{1}{2m^*}\int d^2\mathbf{r}\ \psi^{\dagger}(-\mathrm{i}\boldsymbol{\nabla} + e\mathbf{A} - \mathbf{a}(\mathbf{r}))^2\psi + V \qquad \mathbf{a}(\mathbf{r}) \equiv \tilde{\phi}\int d^2\mathbf{r}'\frac{\hat{\mathbf{z}}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\rho(\mathbf{r}')$$

  - Fictitious magnetic field: $\mathbf{b}(\mathbf{r}) \equiv \boldsymbol{\nabla}\times\mathbf{a}(\mathbf{r}) = 2\pi\rho(\mathbf{r})\tilde{\phi}$
  - Fictitious and external (real) B-field cancel **on average** at exactly ν = 1/2

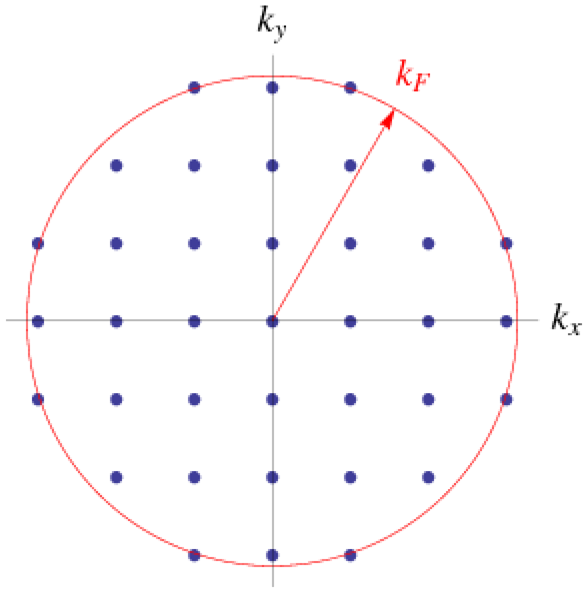
$$B_{\mathrm{eff}} = B - \langle b(\mathbf{r})\rangle = B - 2\pi\tilde{\phi}n_{\mathrm{e}}$$

Composite fermion "theory"!

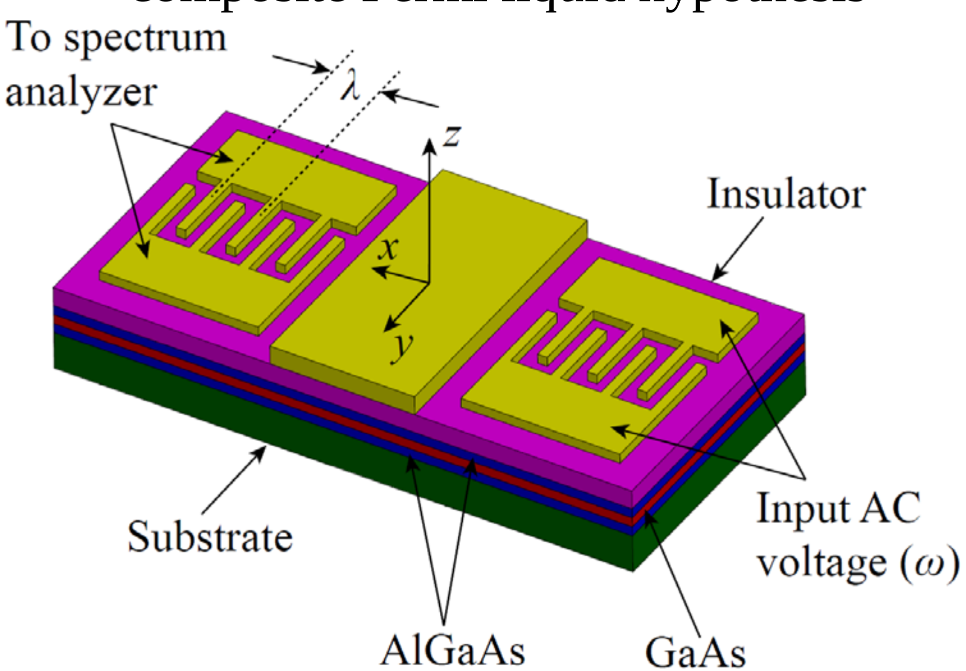
- **Experimental verification**
  - Most important parameters of a Fermi liquid?

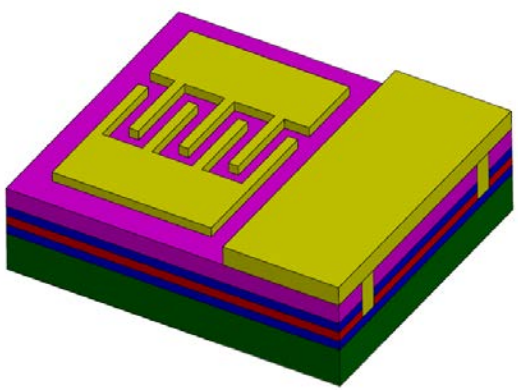
    $k_{\mathrm{F}} = \sqrt{4\pi n_e}$ $\qquad m_{\mathrm{eff}}$
  - Testing biggest hypothesis: do quasiparticles really see B_{eff}?

    $B_{\mathrm{eff}} = B - B_{\nu=1/2}$



  - Yes! Surface acoustic wave (SAW) experiment on $\nu = 1/2$ prove composite Fermi liquid hypothesis

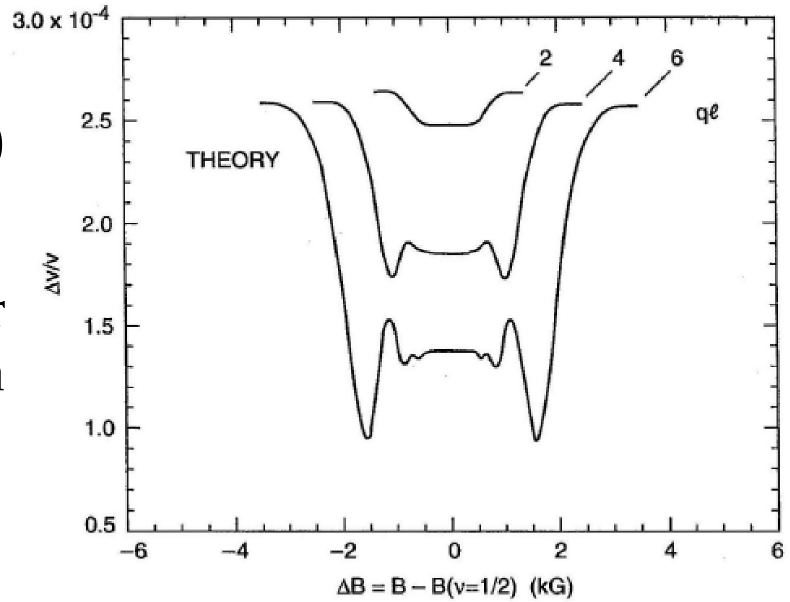


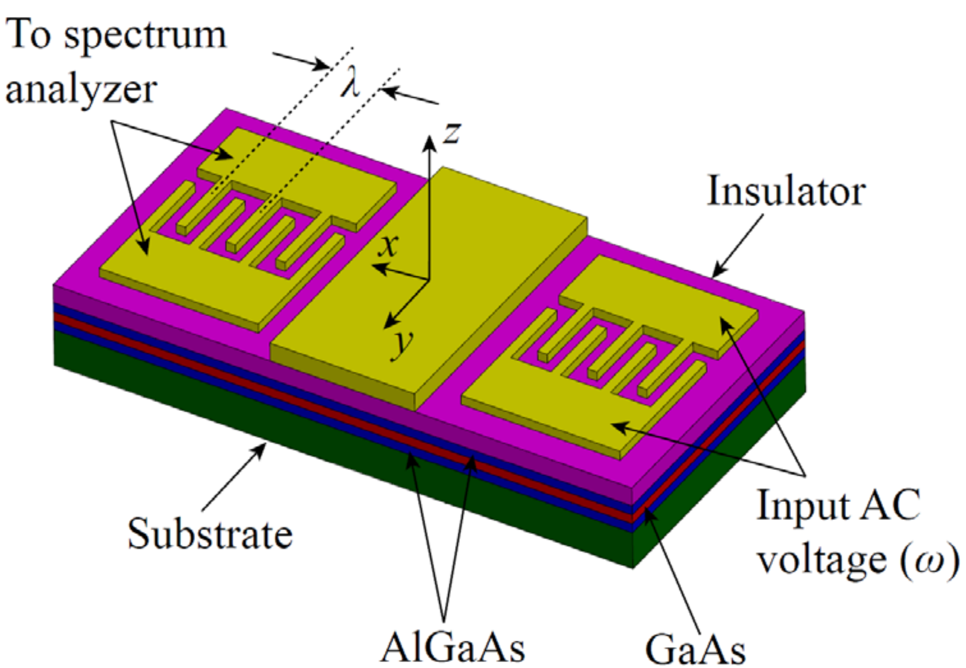


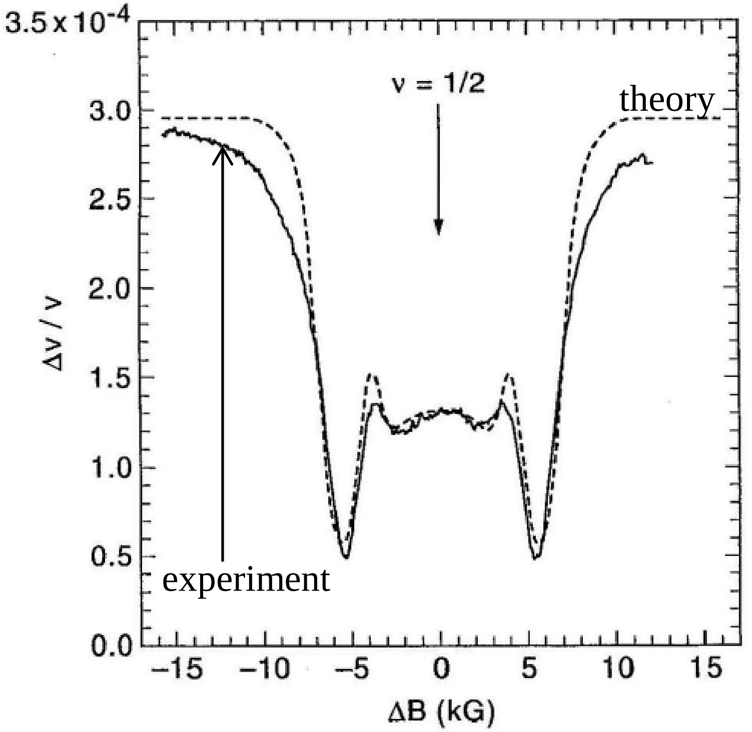
Composite fermion "theory"!

- **Surface acoustic wave (SAW) propagation experiment**
  - HLR predicted SAW resonance for probe wavelength less than CF mean free path (ℓ)

$$\frac{\Delta v}{v} = \left(\frac{\alpha^2}{2}\right)\frac{1}{1+\left(\frac{\sigma_{xx}(\omega,q)}{\sigma_{\mathrm{m}}}\right)^2} \qquad q\ell \gg 2$$



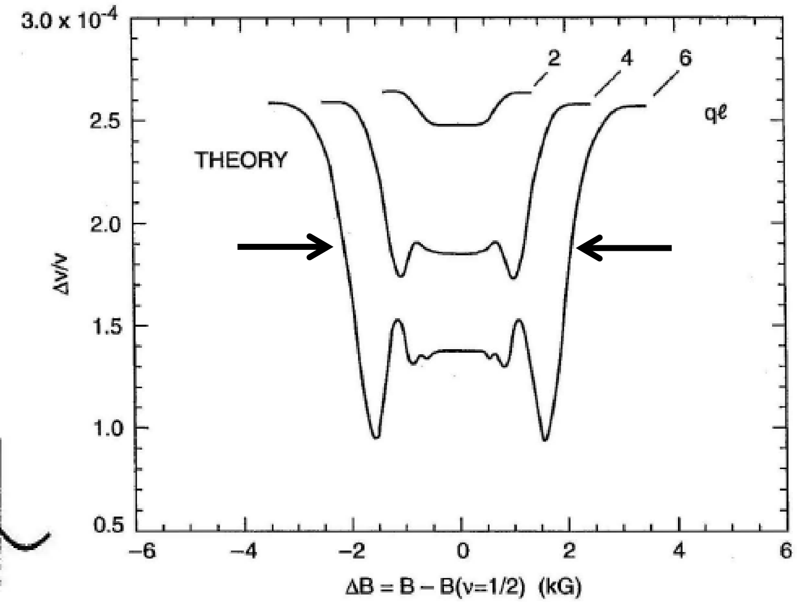




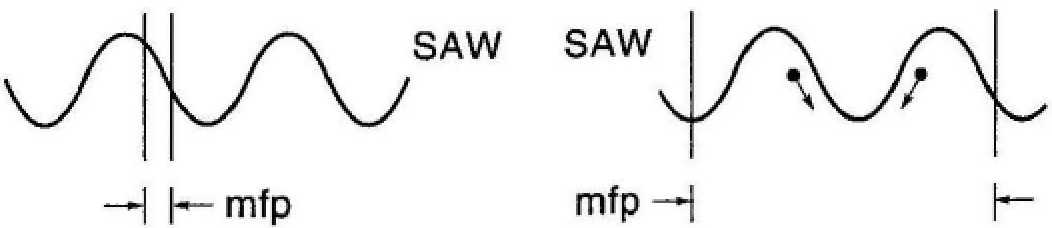
Composite fermion "theory"!

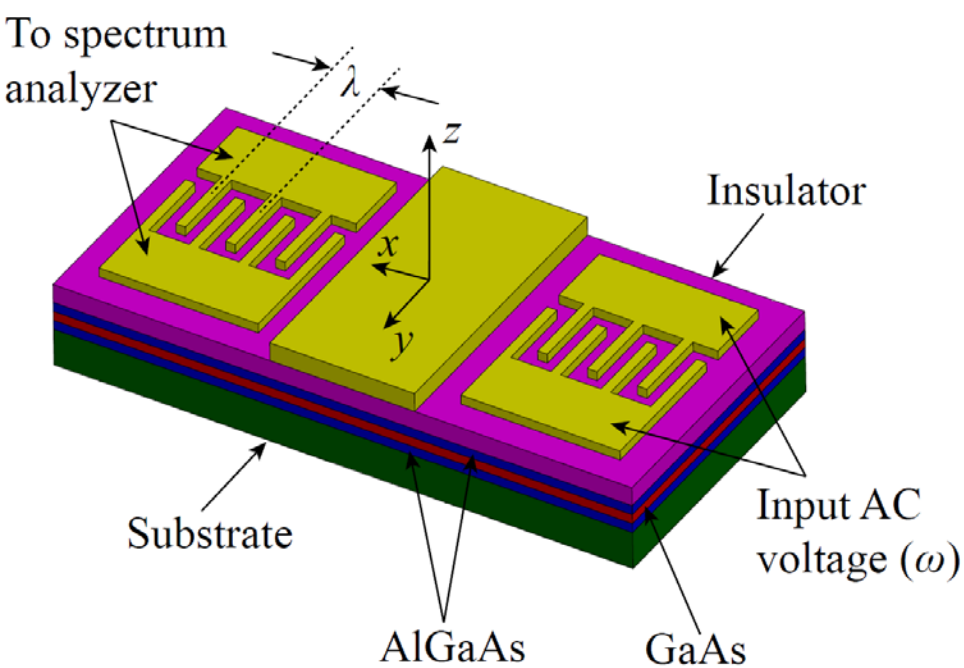
- **SAW interpretation**

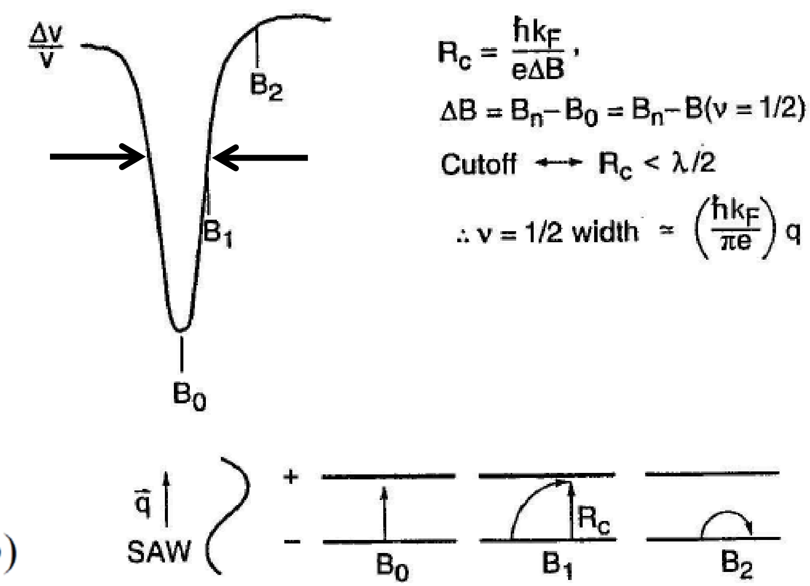
$$\frac{\Delta v}{v} = \left(\frac{\alpha^2}{2}\right)\frac{1}{1+\left(\frac{\sigma_{xx}(\omega,q)}{\sigma_{\mathrm{m}}}\right)^2} \qquad \boxed{q\ell \gg 2}$$



$q\ell < 2$ $\qquad$ $q\ell > 2$



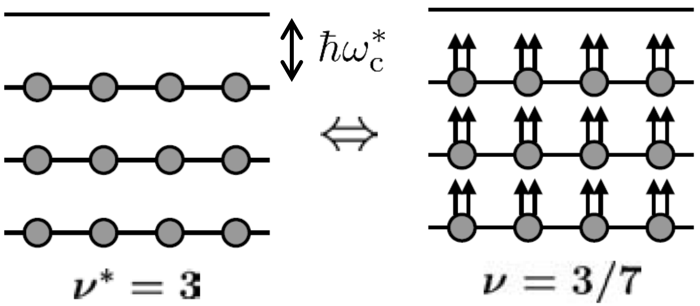




Composite fermion “theory”!

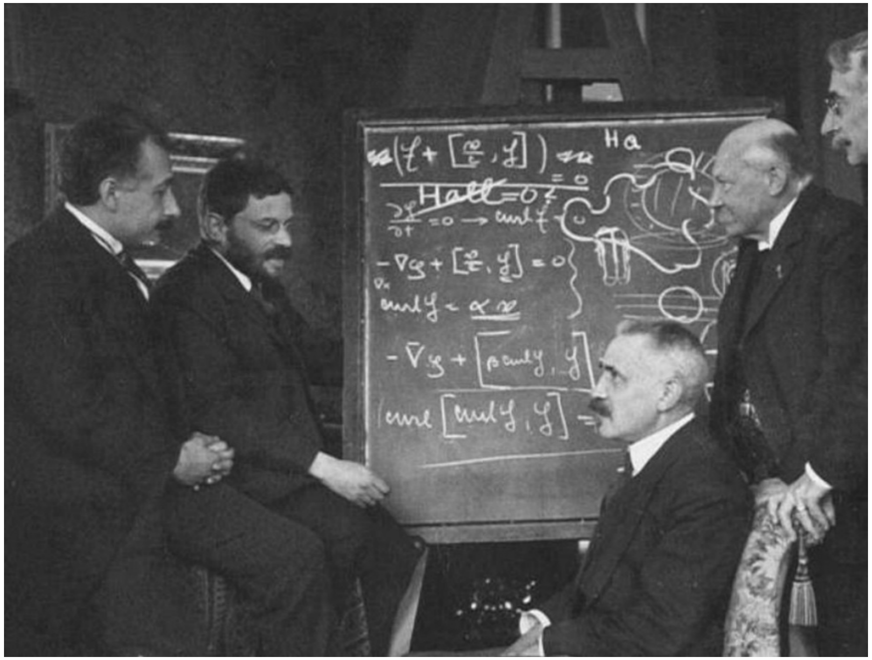
- **Mass of a composite fermion**
  - Cyclotron gap between Λ levels?

$$\Delta = \hbar\omega_c^* = \frac{e\hbar\Delta B}{m^*}$$

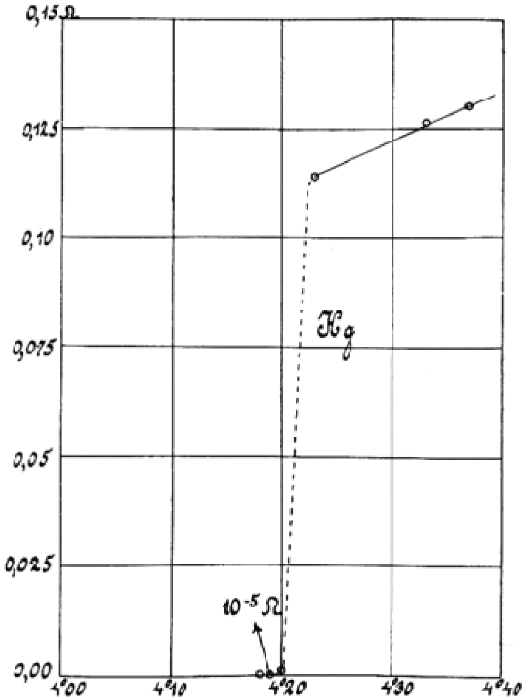


Odd-denominator states as "composite bosons"

- **Non-composite fermion flux attachment schemes**
  - Heike Kamerlingh Onnes produces liquid ^{4}He on 10 July, 1908
  - On 8 April, 1911 he discovered superconduct-ivity in a solid Hg wire at 4.2 K
  - Quantum origins of superconductivity a mystery until 1957

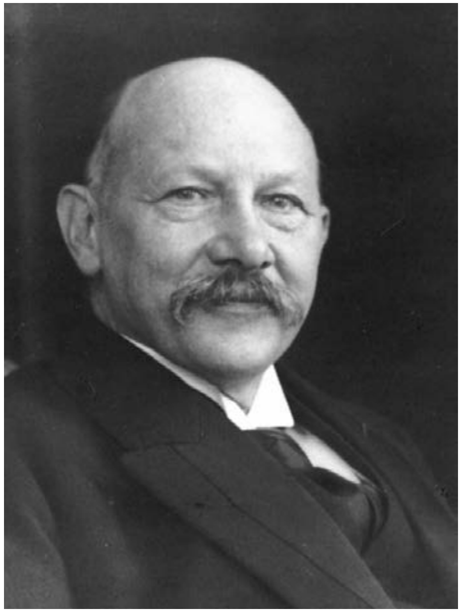


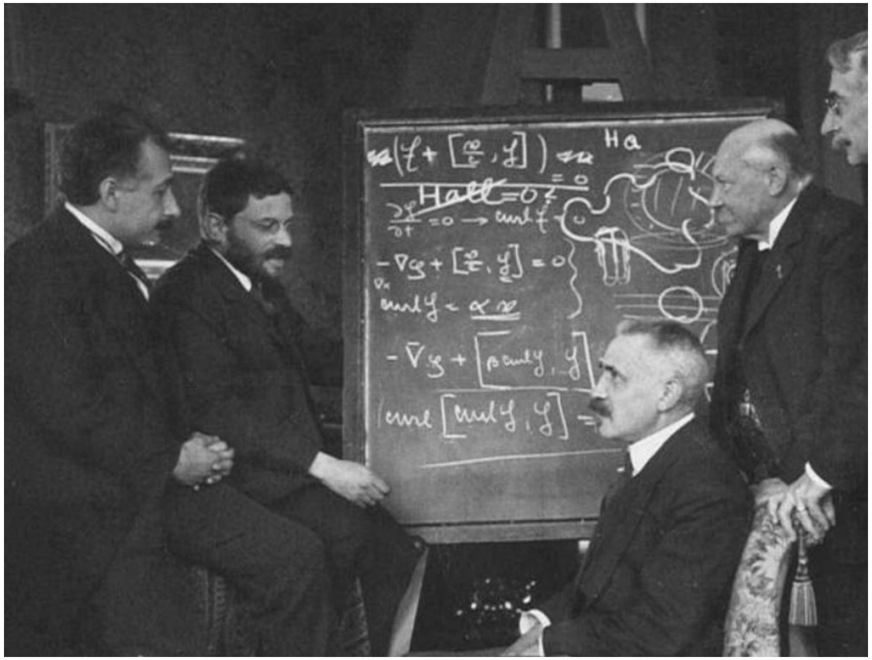
Einstein, Ehrenfest, Langevin, Kamerlingh Onnes, and Weiss at a workshop in Leiden October 1920. The blackboard discussion, on the Hall effect in superconductors



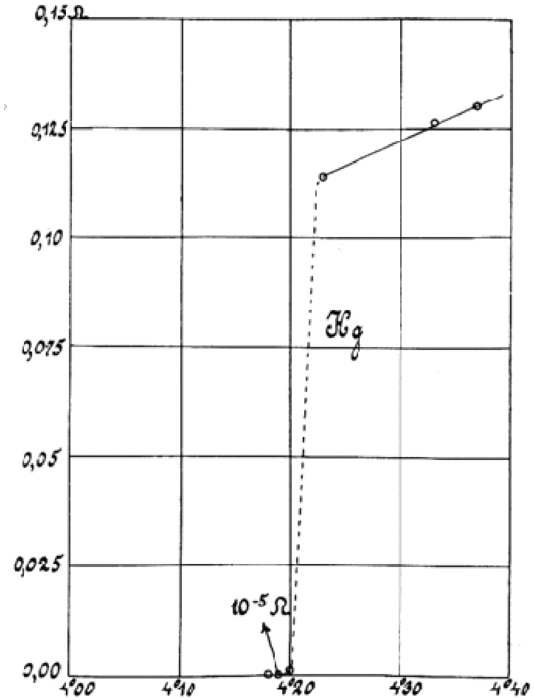
Closing remarks: spin DOF?

- **QHE in GaAs**
  - Heike Kamerlingh Onnes produces liquid ^{4}He on 10 July, 1908
  - On 8 April, 1911 he discovered superconduct-ivity in a solid Hg wire at 4.2 K
  - Quantum origins of superconductivity a mystery until 1957





Einstein, Ehrenfest, Langevin, Kamerlingh Onnes, and Weiss at a workshop in Leiden October 1920. The blackboard discussion, on the Hall effect in superconductors

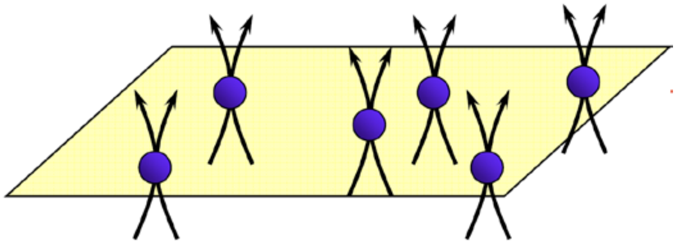


Closing remarks: edge states?

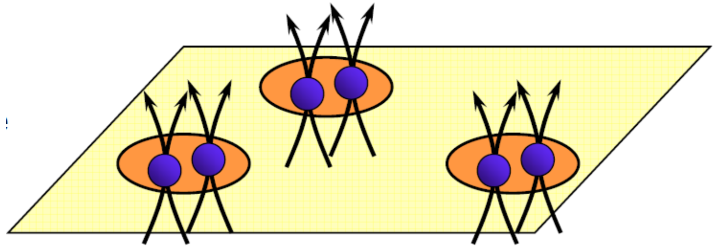
- **Composite Fermi liquid or Luttinger liquid?**
  - Heike Kamerlingh Onnes produces liquid ^{4}He on 10 July, 1908
  - On 8 April, 1911 he discovered superconduct-ivity in a solid Hg wire at 4.2 K
  - Quantum origins of superconductivity a mystery until 1957

Next time: even-denominator "plateaus"

- **Moore-Read state ($\nu = 5/2 = 2 + 1/2$)**
  - Effects of composite fermion formation
    - Absorb flux ⇒ metal in zero B-field at even denominators
    - Absorb interactions ⇒ mass of composite fermions



  - Residual B-field ⇒ IQHE of composite fermions ⇒ odd denominators
  - Residual interactions ⇒ back to intractable interacting problem?
  - Attractive interactions? ⇒ BCS instability?



Thanks for listening

Questions?