Heavy Fermion & Kondo Insulator

Min-Feng

- What is heavy fermion
- What is Kondo insulator
- How to study Kondo insulator

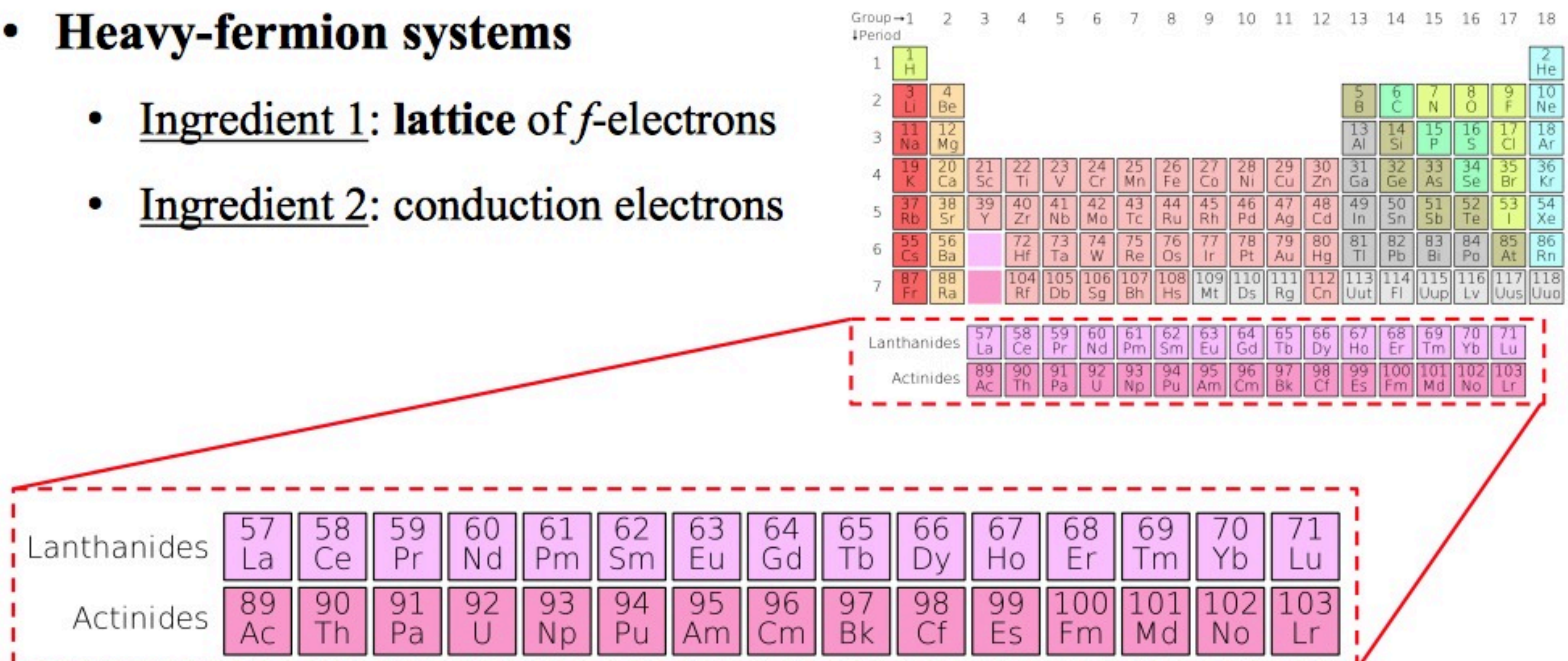
What is heavy fermion

Definition

- What?

  a specific type of inter-metallic compound, containing elements with 4f or 5f electrons.

- **Heavy-fermion systems**
  - Ingredient 1: **lattice** of *f*-electrons
  - Ingredient 2: conduction electrons



Definition

- What?
  a specific type of inter-metallic compound, containing elements with 4f or 5f electrons.

- Why?
  The low-temperature specific heat whose linear term is up to 1000 times larger than the value expected from the free-electron theory...

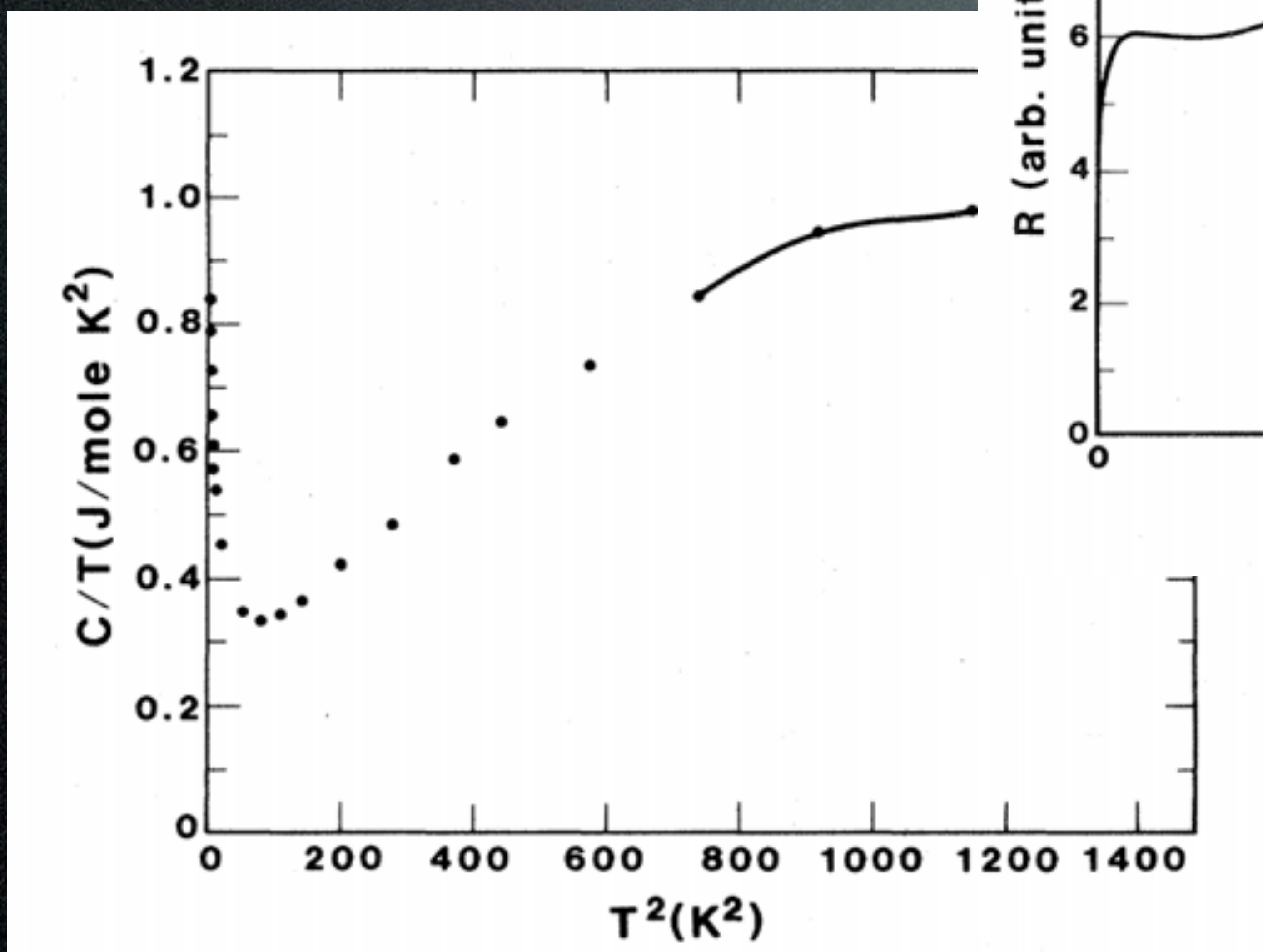
$$C_v = C_{el} + C_{ph} + C_{others}$$

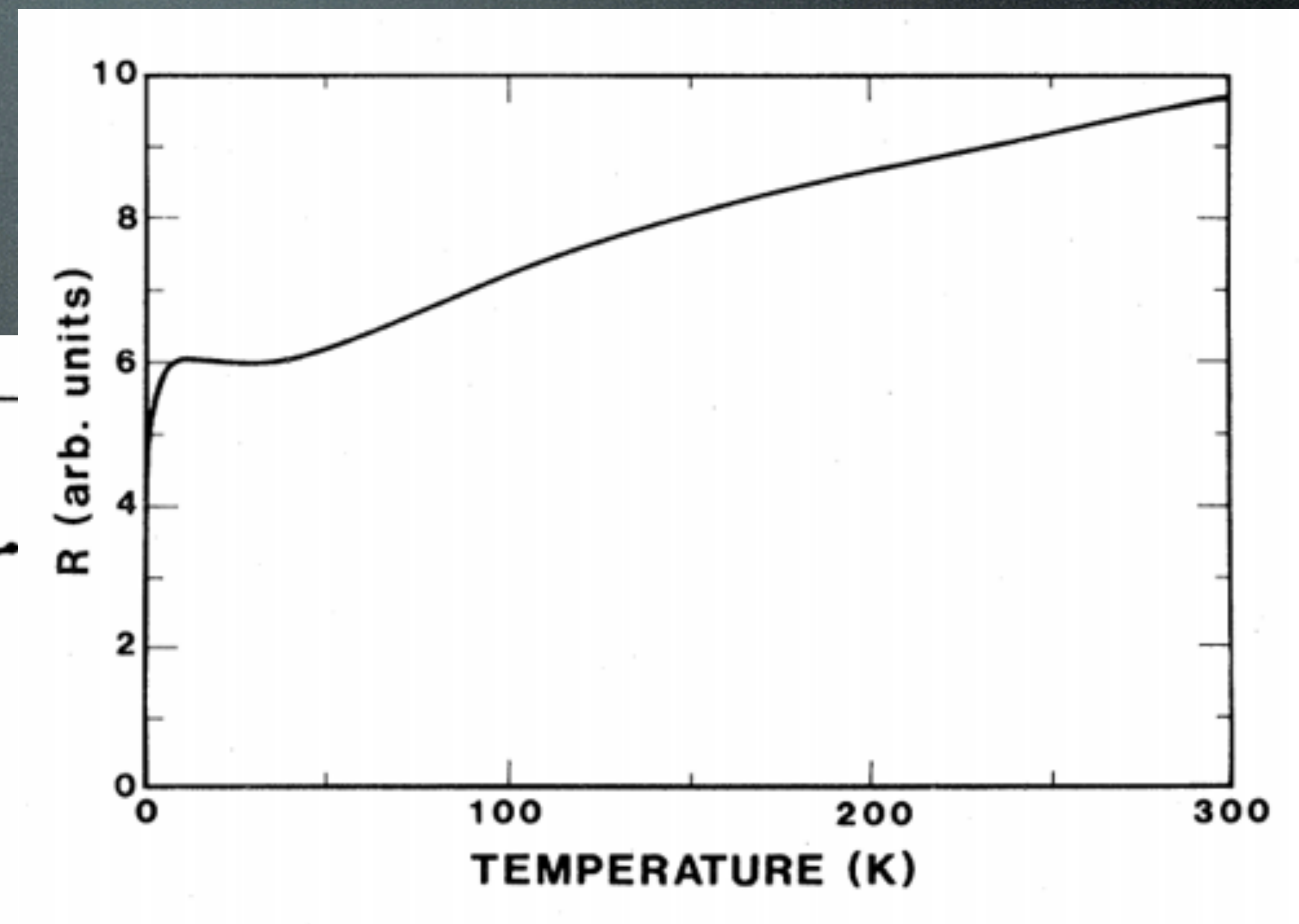
$$C_{el} \propto \gamma T$$

$$\gamma \propto m_{eff}$$

Example

Metal $CeCu_6$





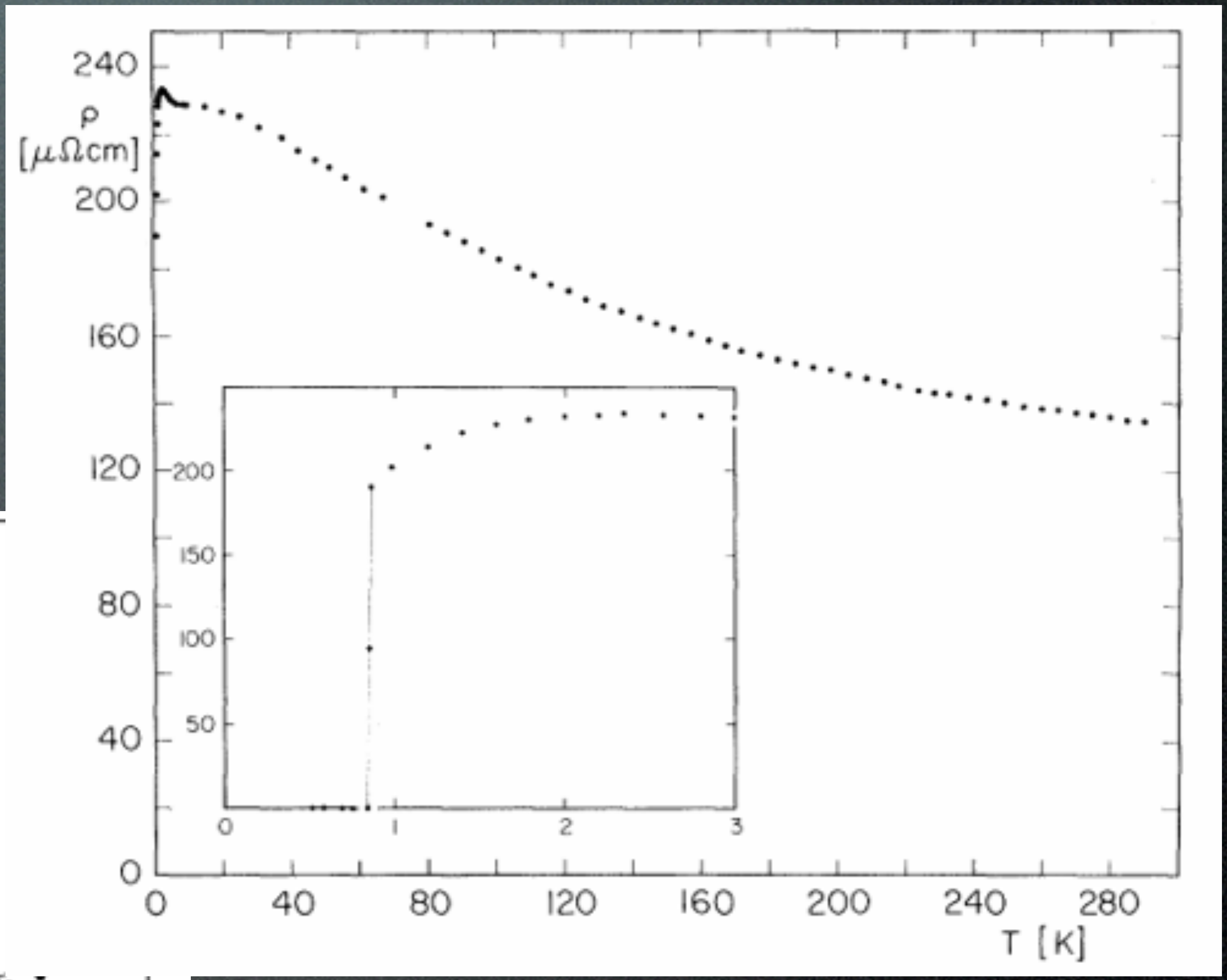
$T = 10K$

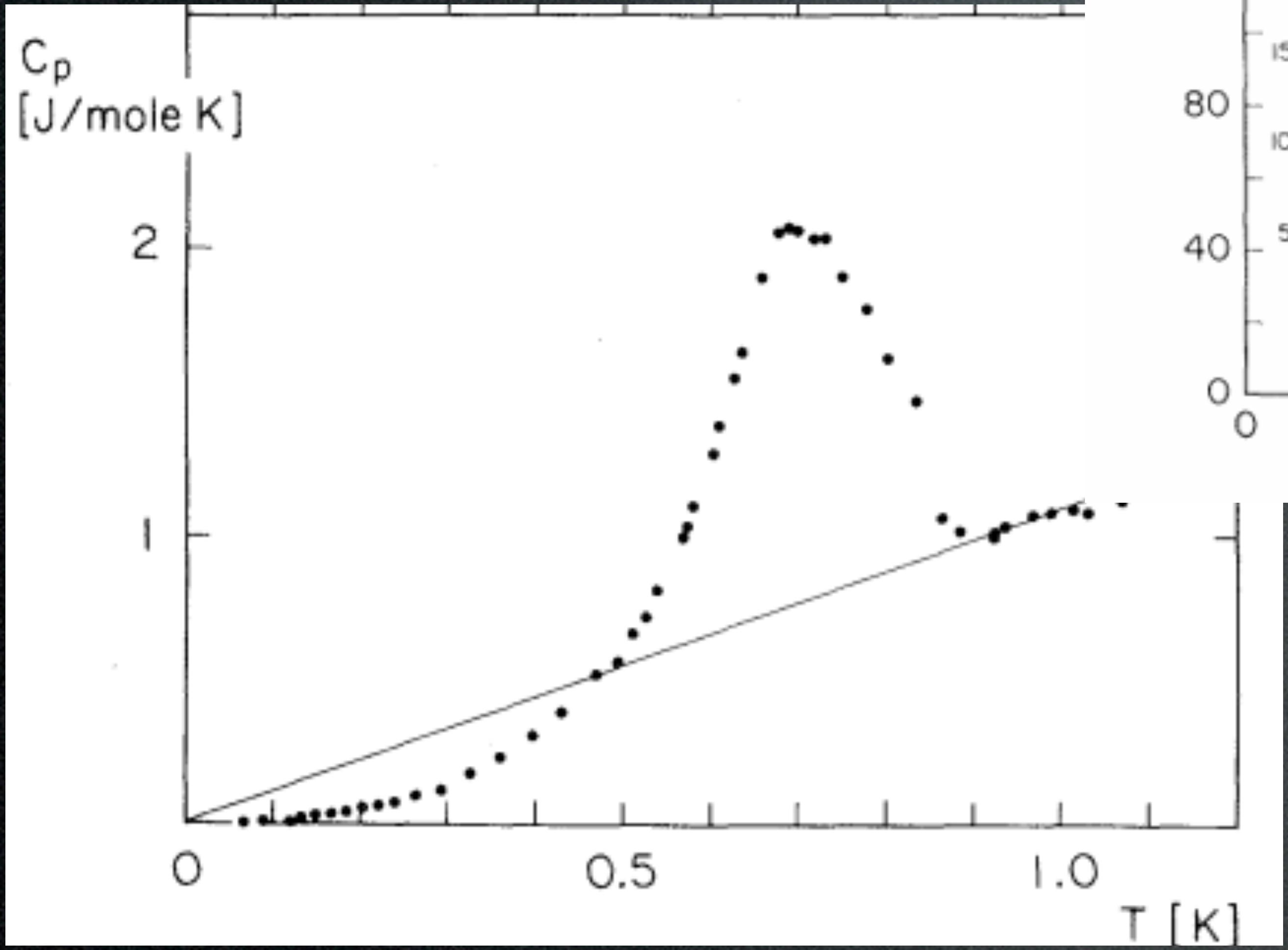
$\gamma = 1600 mJ mol^{-1} K^{-2}$

$R \propto T^2$

Example

SC UBe_{13}



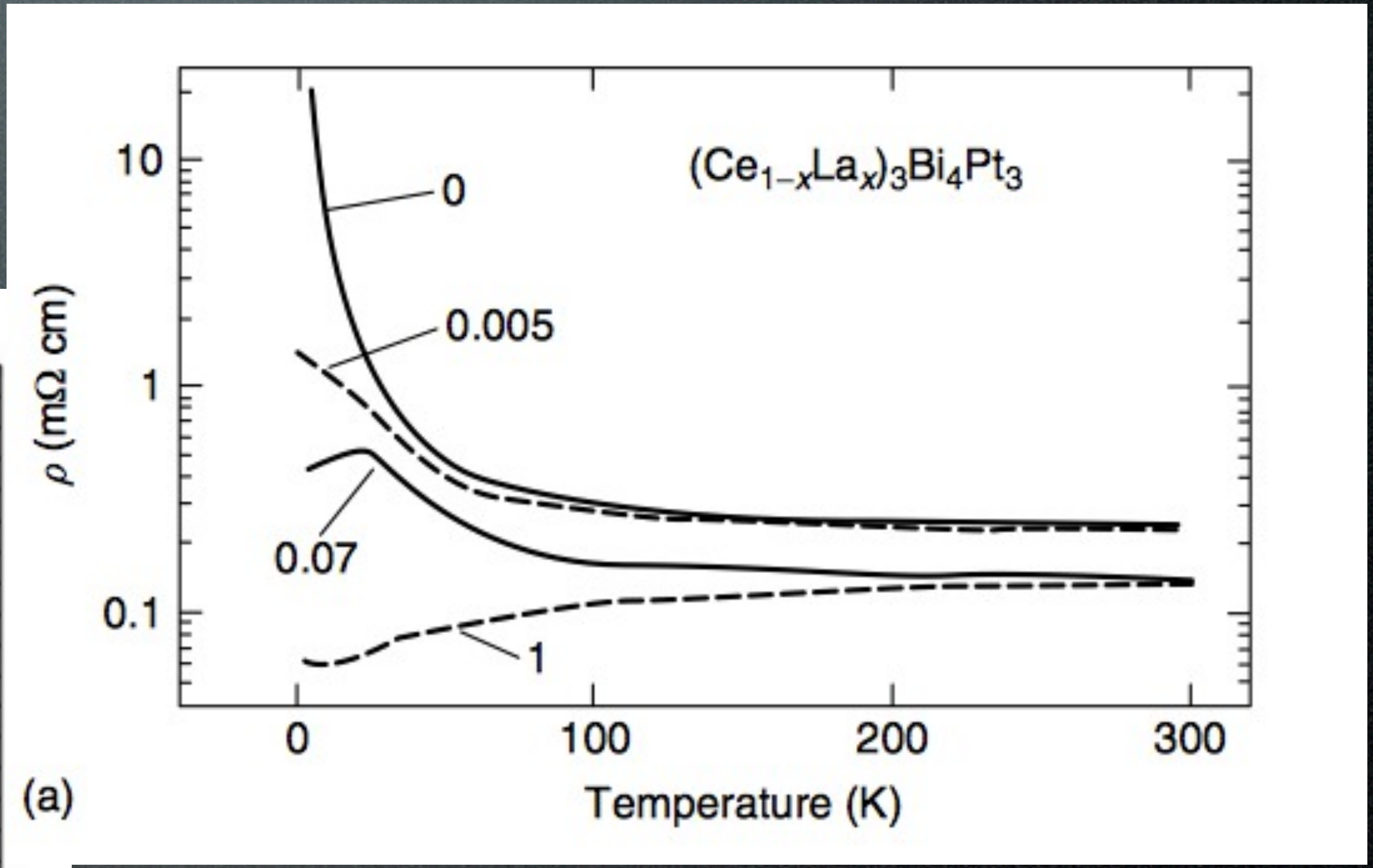


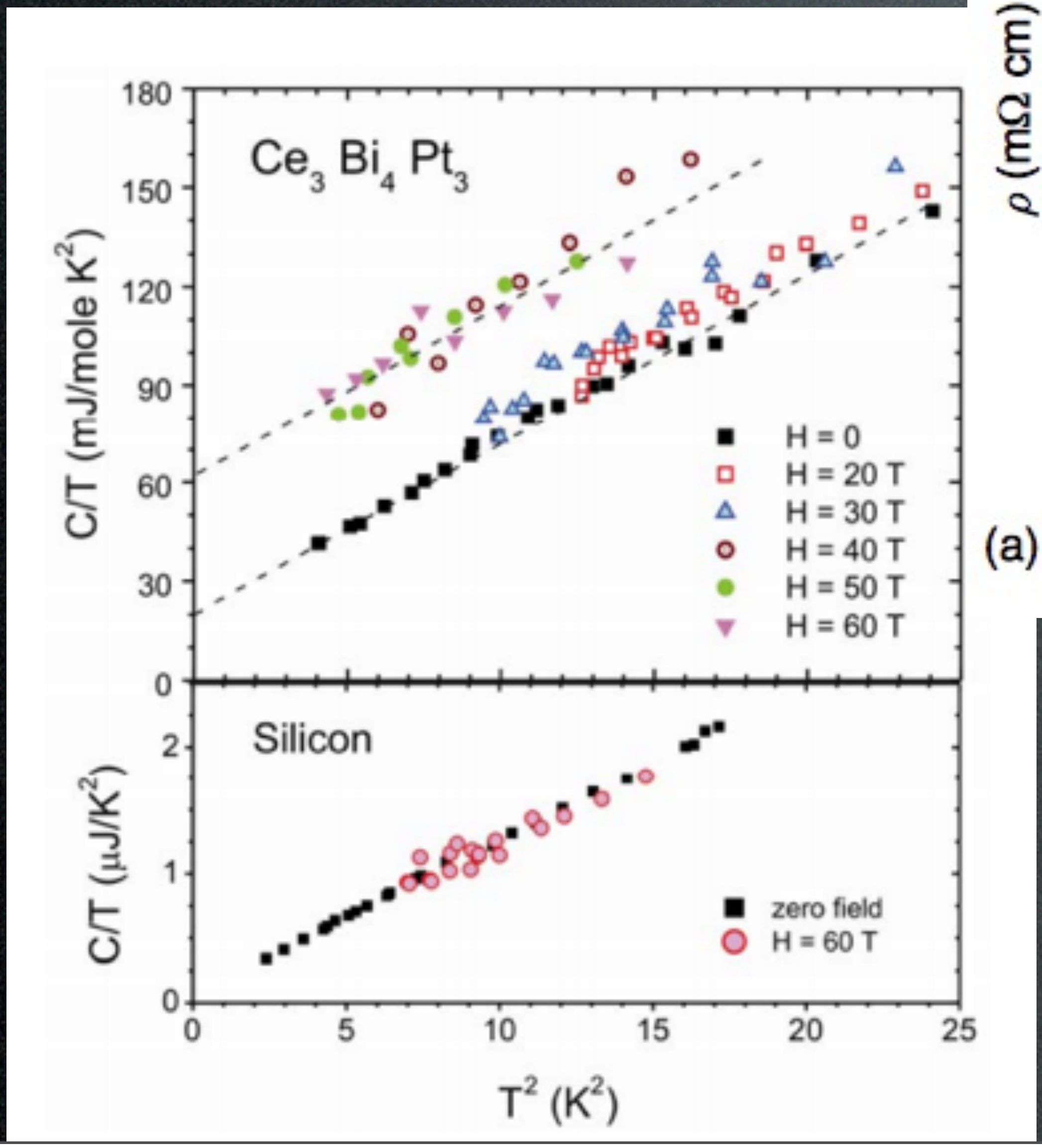
$$T = 0.8K$$
$$\gamma = 800mJmol^{-1}K^{-2}$$

Example

Kondo Insulator

$Ce_3Pt_4Bi_3$





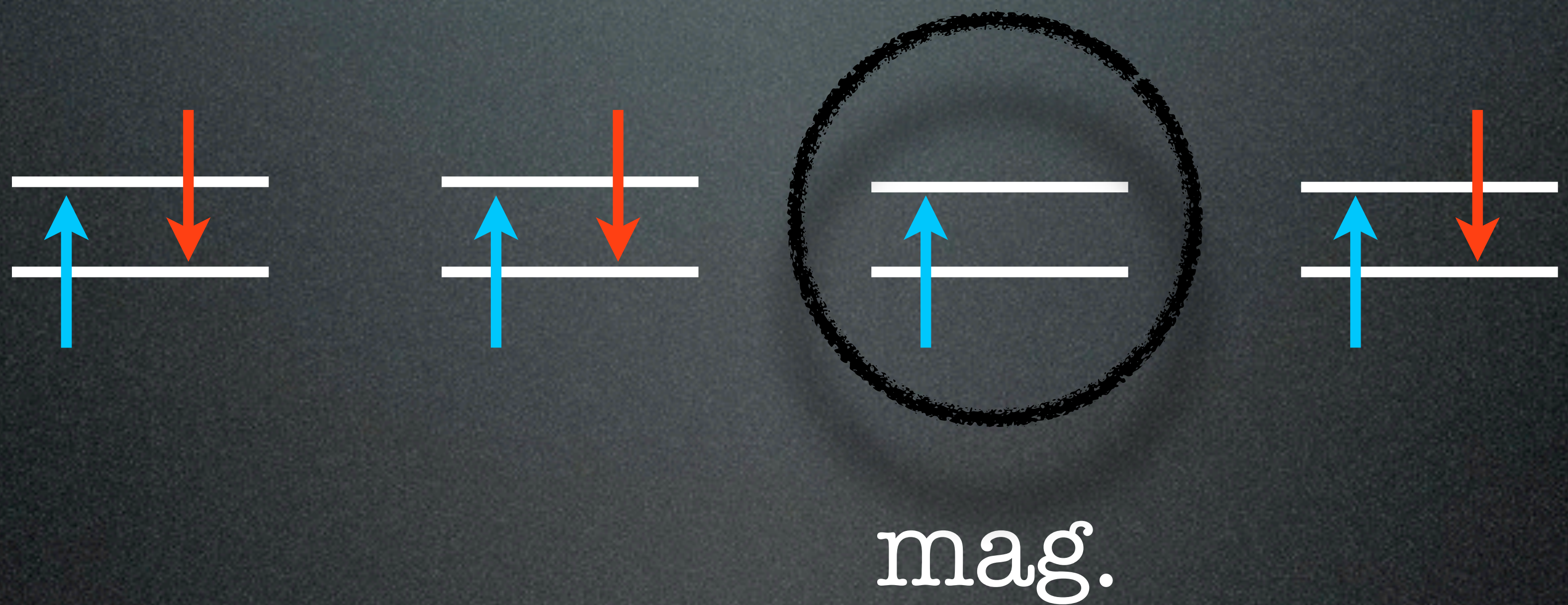
$T_\chi = 80K$

$\gamma \approx low$

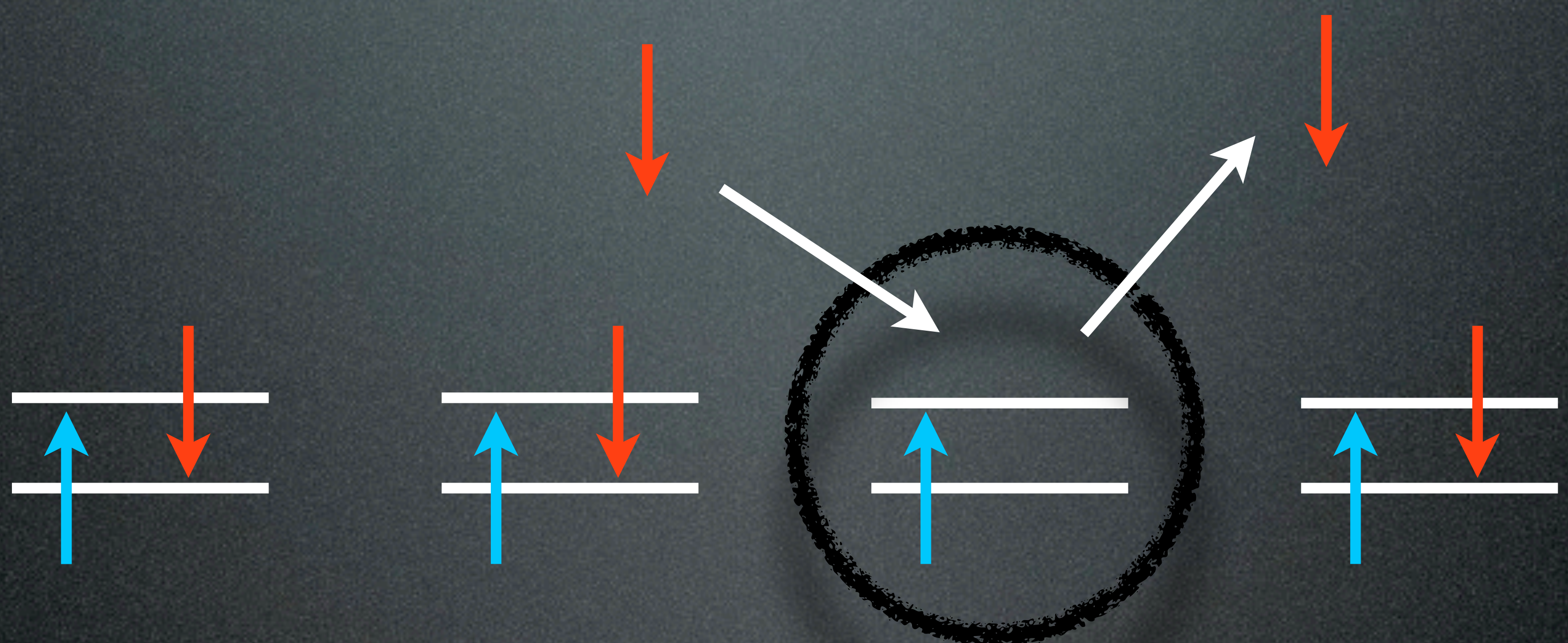
insulator has low election conduction

Brief Explanation

- Partially filled 4f or 5f

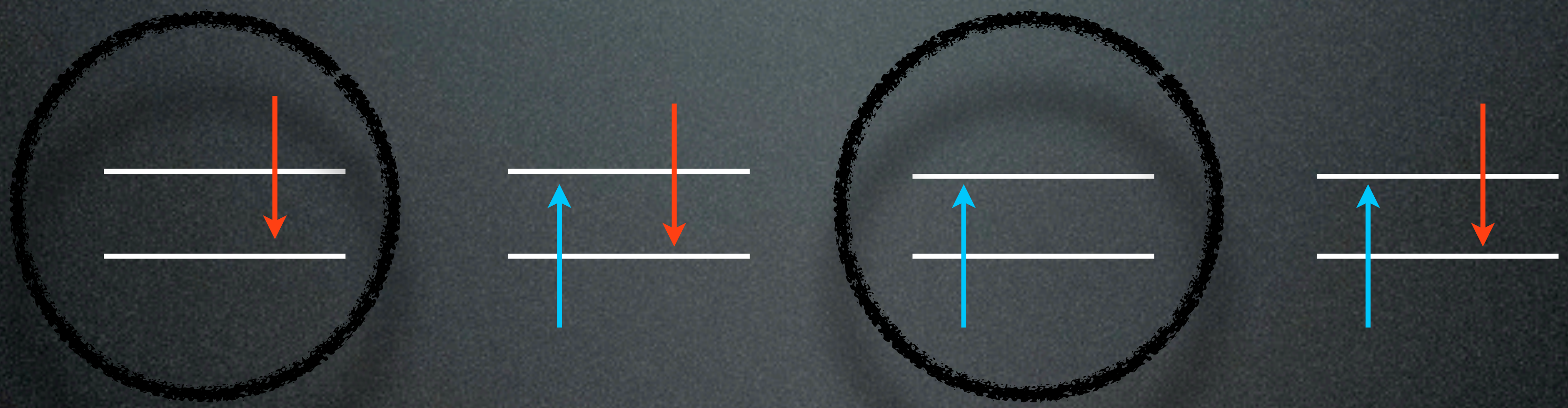


Brief Explanation



scattering

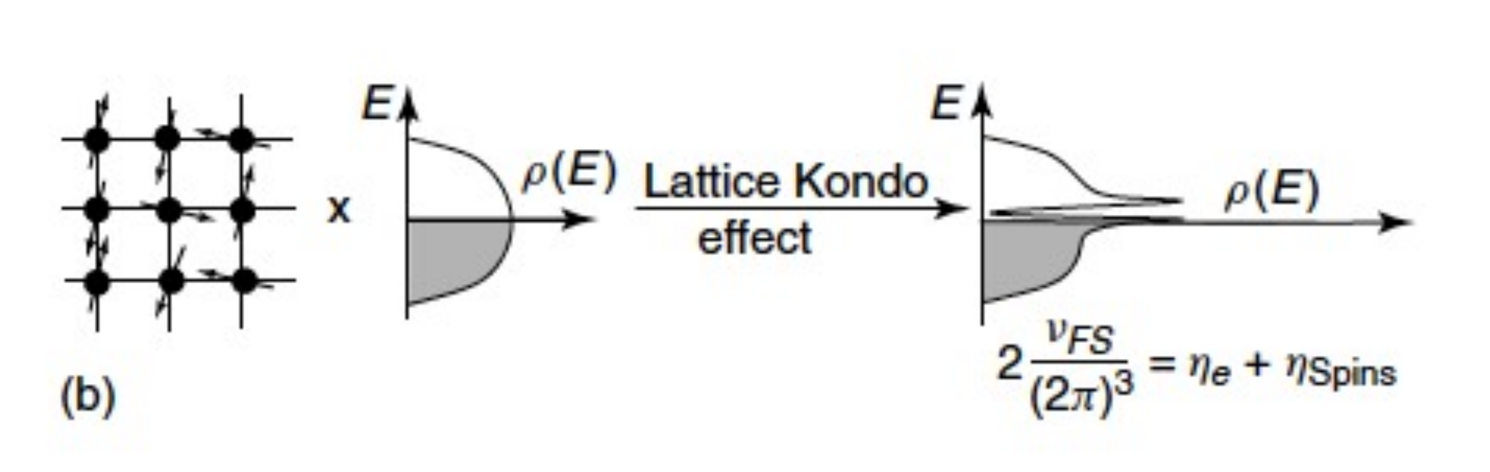
Brief Explanation



Coherence or RKKY

What is Kondo insulator

Definition



- What?
  One of the dense lattice of local moments phenomena.

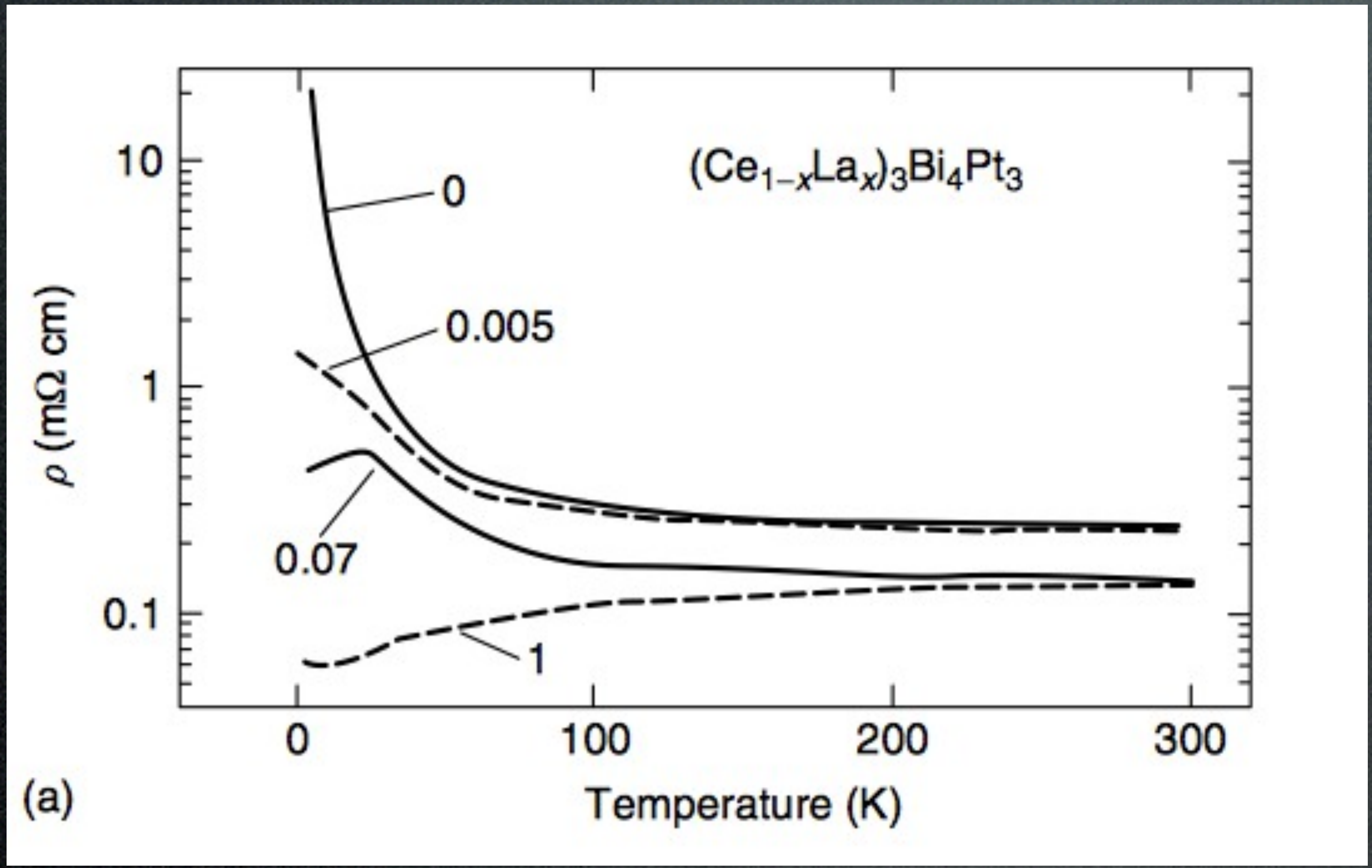
- Important response as T decreases?

$$R \rightarrow increase$$
$$\chi \rightarrow decrease$$

Brief Explanation

- What is the difference between KI and normal insulator?

Each Kondo insulator has its fully itinerant semiconductor

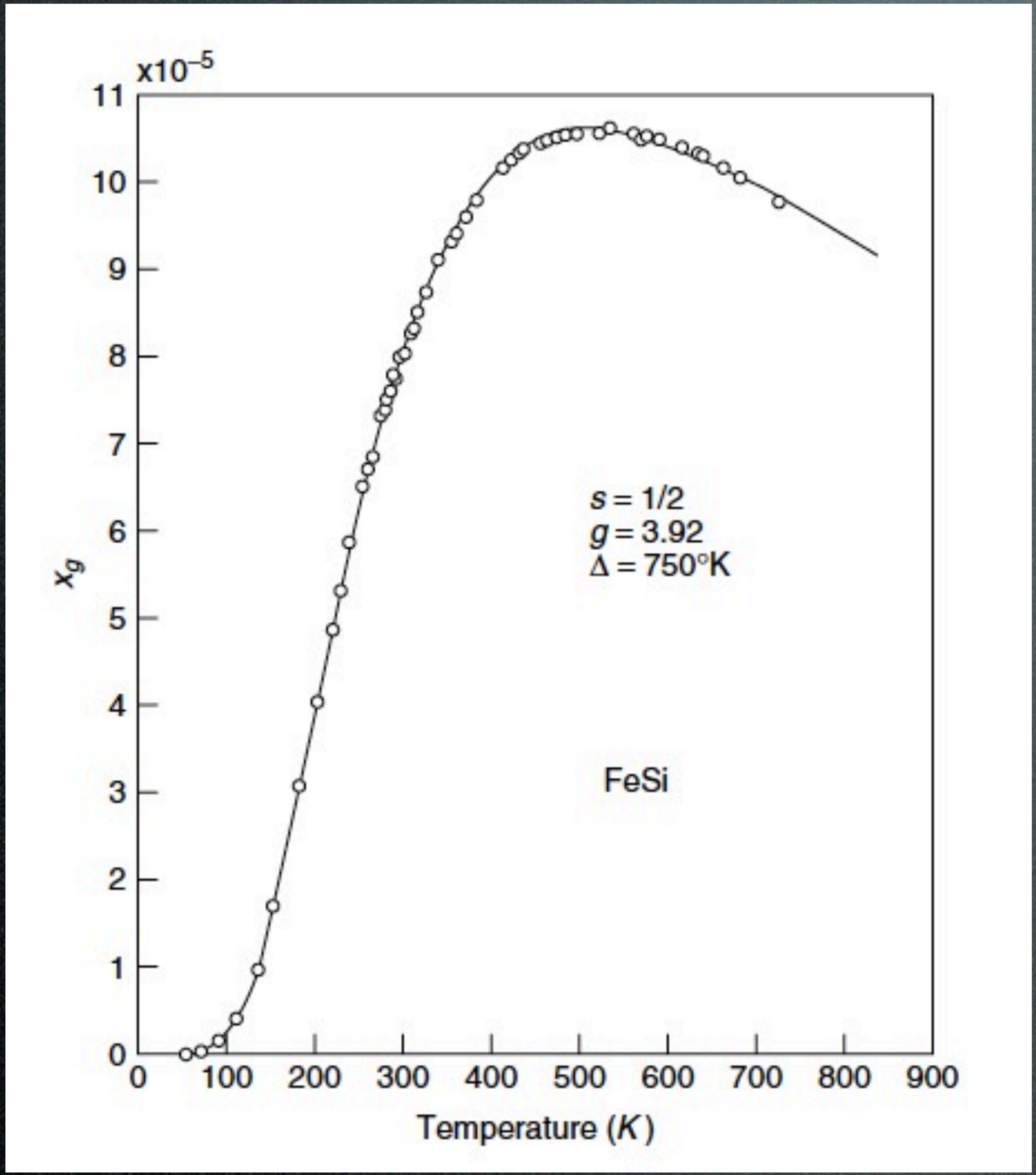
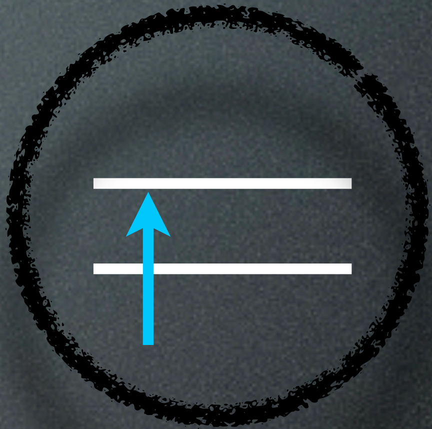
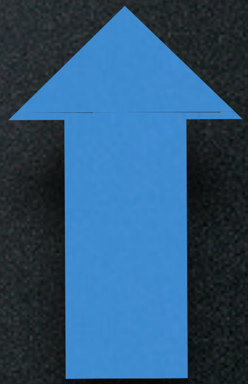


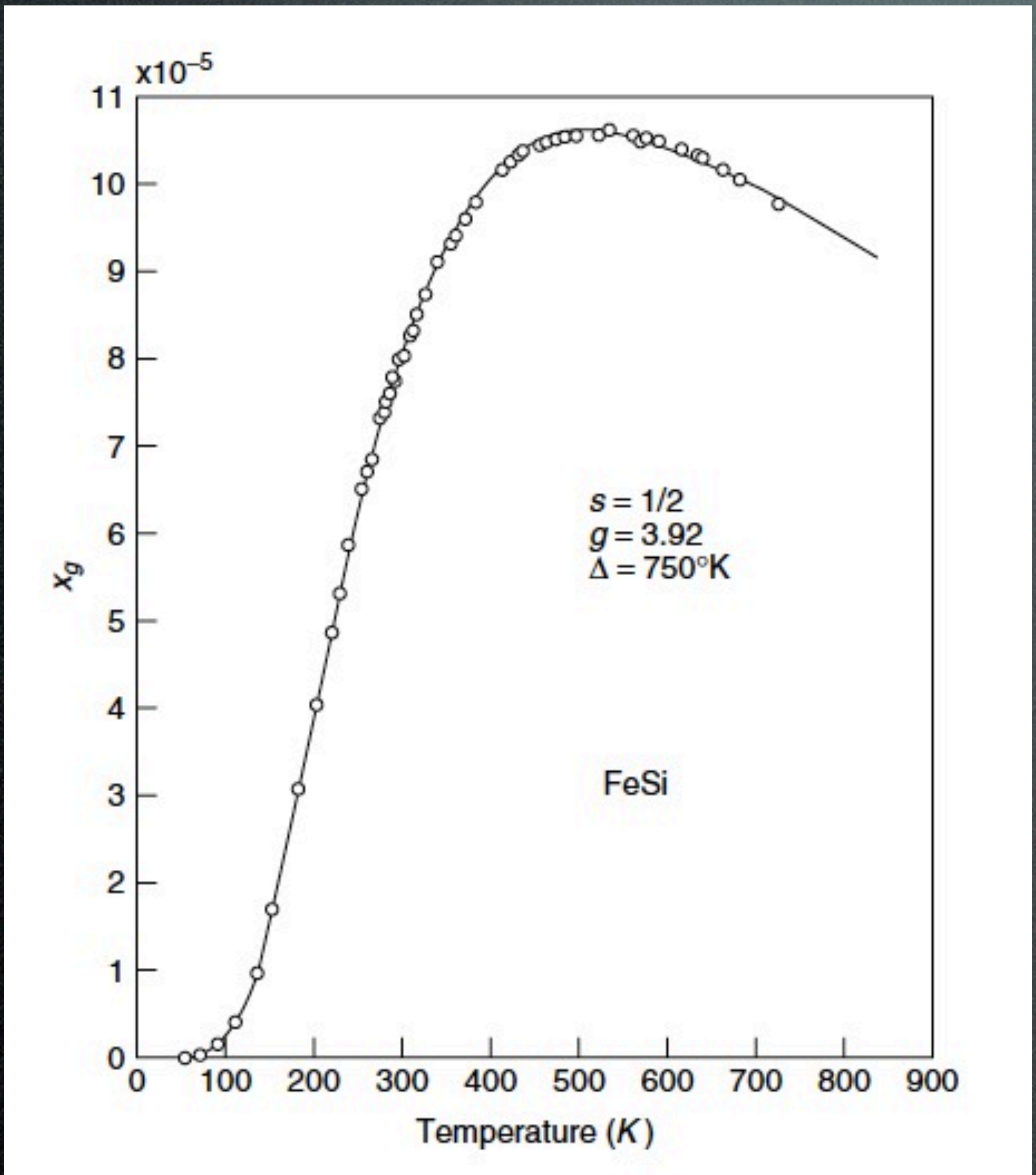
$CeBi_4Pt_3$

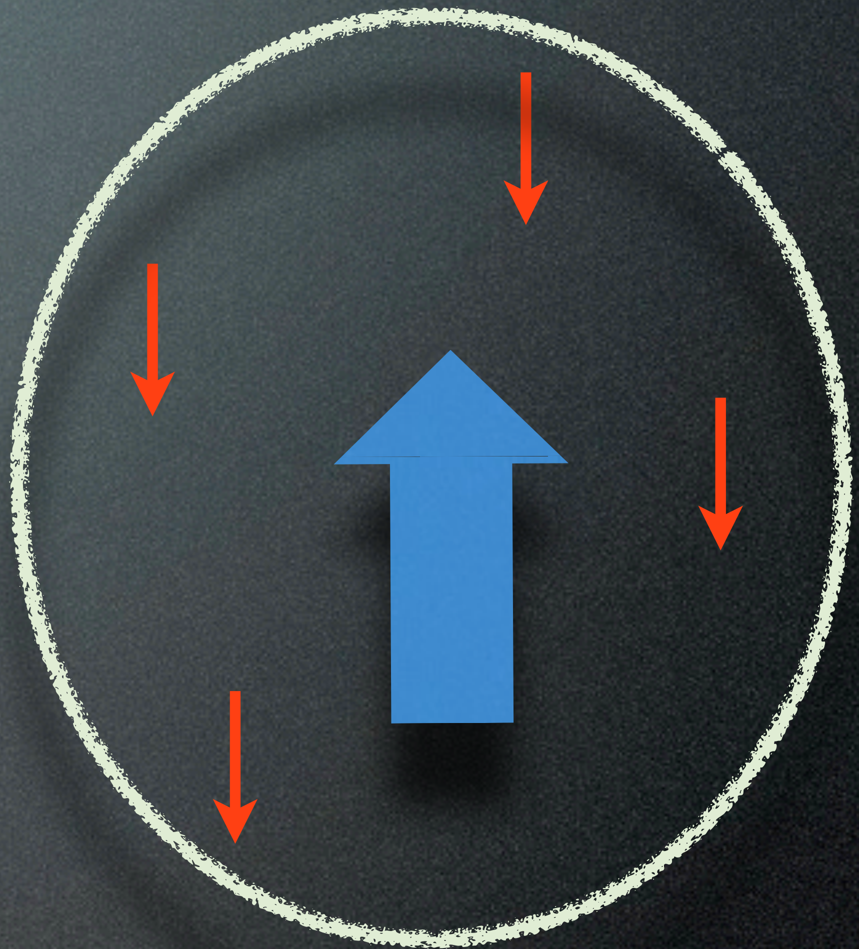
KI

$LaBi_4Pt_3$

normal


x10−5
11
10
9
8
7
6
5
4
3
2
1
0
χ_g
0
100
200
300
400
500
600
700
800
900
Temperature (K)
s = 1/2
g = 3.92
Δ = 750°K
FeSi







non-magnetic

How to study the Kondo insulator

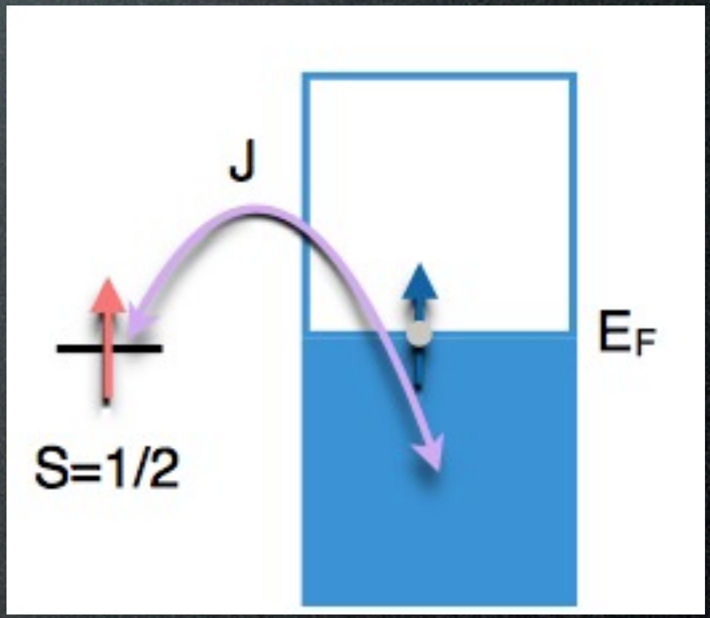
Theoretical Method

- Anderson model
- Kondo lattice model
- Many others....

Anderson Model

The idea: partially filled states + conduction electrons

$$H = \overbrace{\sum_{k,\sigma} \epsilon_k n_{k\sigma} + \sum_{k,\sigma} V(k) \left[c_{k\sigma}^{\dagger} f_\sigma + f_\sigma^{\dagger} c_{k\sigma} \right]}^{H_{\text{resonance}}} + \underbrace{E_{\text{f}} n_{\text{f}} + U n_{\text{f}\uparrow} n_{\text{f}\downarrow}}_{H_{\text{atomic}}}$$



Atomic term

$$H_{\text{atomic}} = E_f n_f + U n_{f\uparrow} n_{f\downarrow}$$

$$E(f^2) = 2E_f + U$$

$$E(f^1) = E_f$$

$$E(f^0) = 0$$

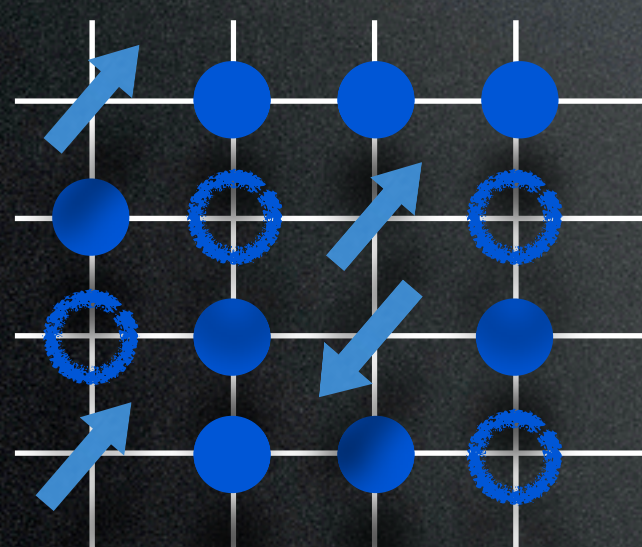
Resonance term

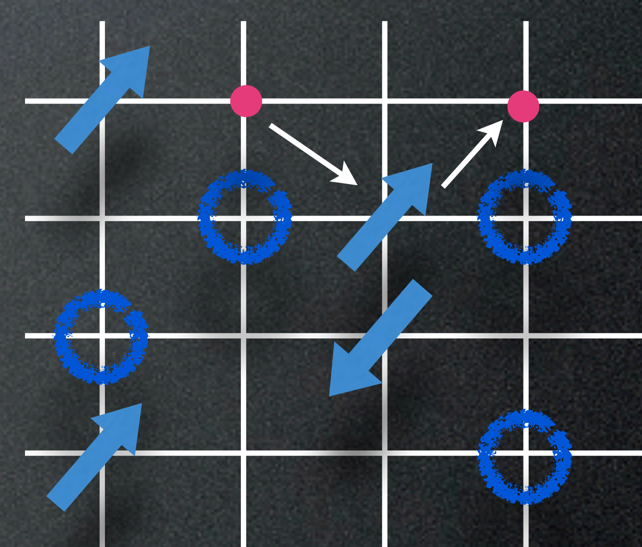
$$H_{\text{resonance}} = \sum_{k,\sigma} \epsilon_k n_{k\sigma} + \sum_{k,\sigma} \left[V(\mathbf{k}) c^{\dagger}_{k\sigma} f_{\sigma} + V(\mathbf{k})^{*} f^{\dagger}_{\sigma} c_{k\sigma} \right]$$

- The competition between these two terms

1. From atomic picture: increase V
   => Always has localized moment

2. From adiabatic picture: increase U
   => Always is a Fermi liquid

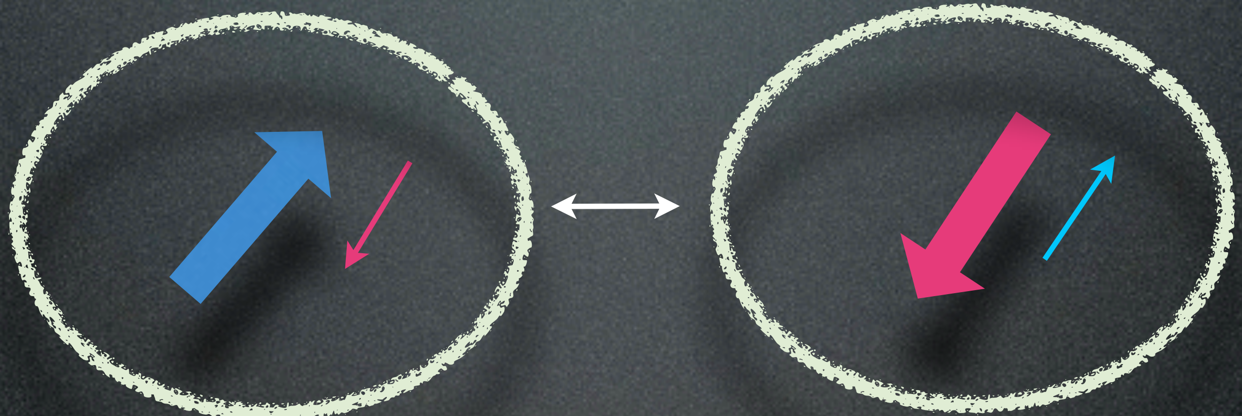




- So how do explain Kondo effect in these model

$$e^-_\downarrow + f^1_\uparrow \rightleftharpoons e^-_\uparrow + f^1_\downarrow$$

The moment tunnels between spin up and spin down, with tunneling rate τ_{sf}



m=0

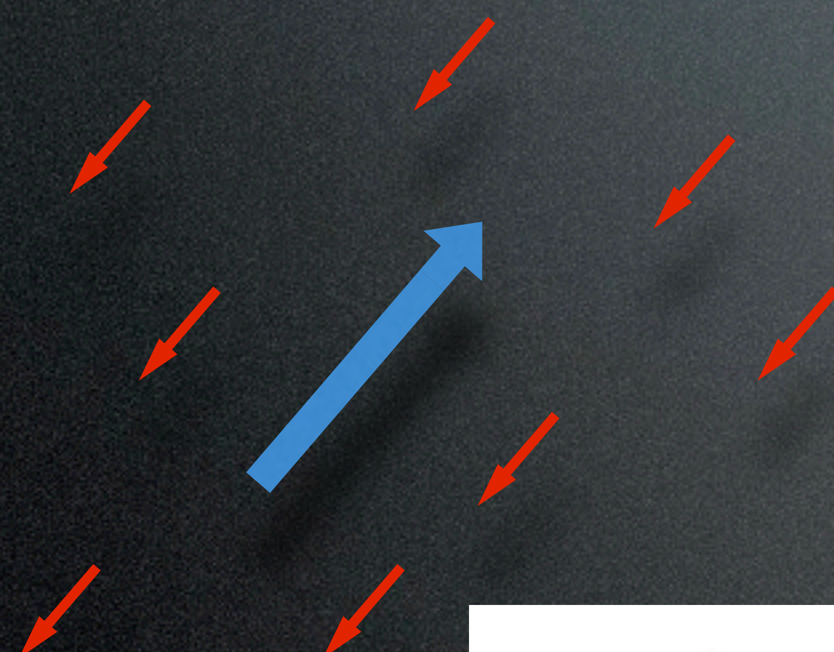
The Kondo state is fixed to the Fermi energy and the resonance is always "on"

requirement $k_B T < k_B T_K = \frac{h}{\tau_{sf}}$

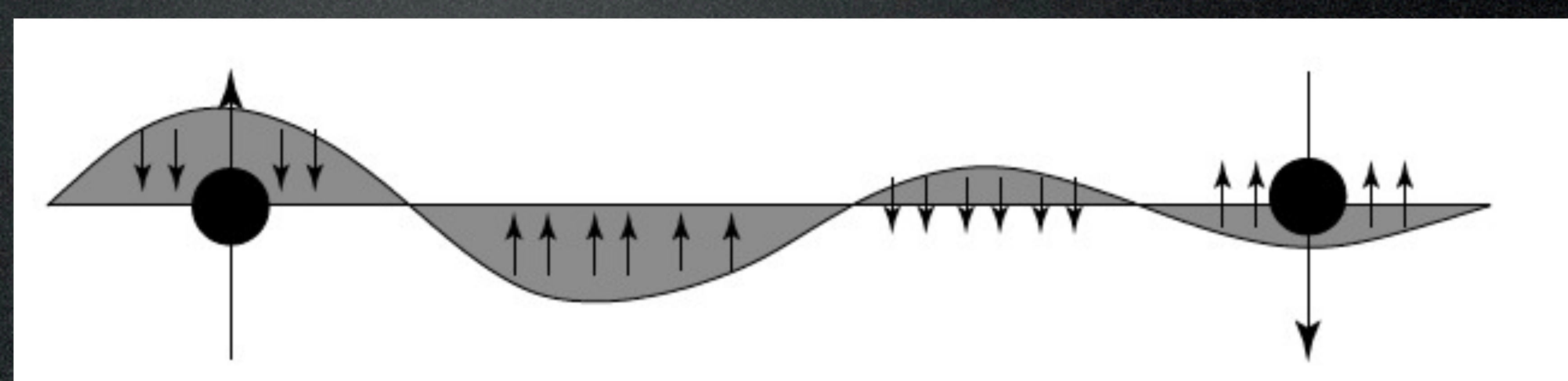
Exchanging frequency is important than the thermal fluctuation

Kondo Lattice model

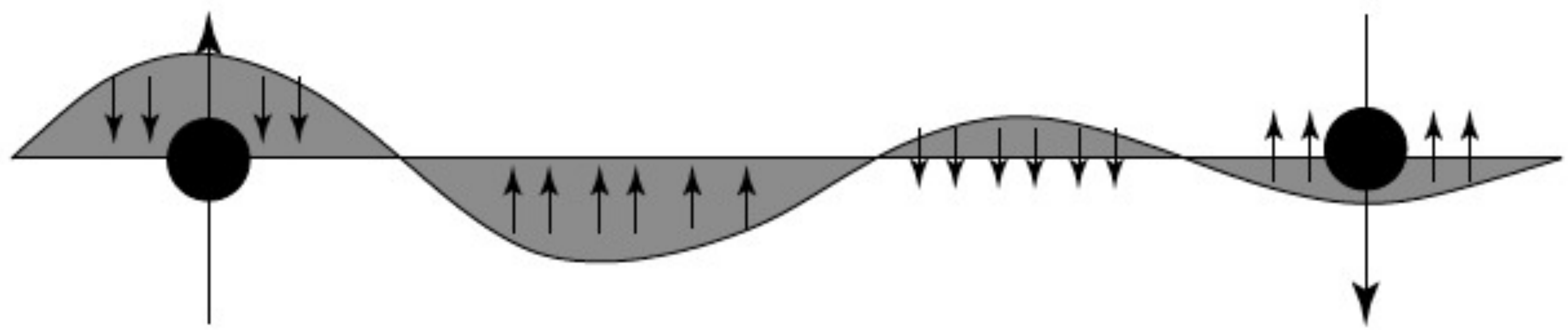
$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \sum_{j} \vec{S}_j \cdot c^{\dagger}_{\mathbf{k}\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{R}_j}$$



Freidel Oscilation



Freidel Oscilation



1. Dilute Kondo effect

$$J \sum_j \vec{S}_j \cdot c^{\dagger}_{\mathbf{k}\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta} \mathrm{e}^{i(\mathbf{k}'-\mathbf{k})\cdot \mathbf{R}_j}$$

$$\langle \vec{\sigma}(r) \rangle \sim -J\rho \frac{\cos 2k_{\mathrm{F}}r}{|k_{\mathrm{F}}r|^3}$$

$$T_{\mathrm{K}} = D\mathrm{e}^{-1/(2J\rho)}$$

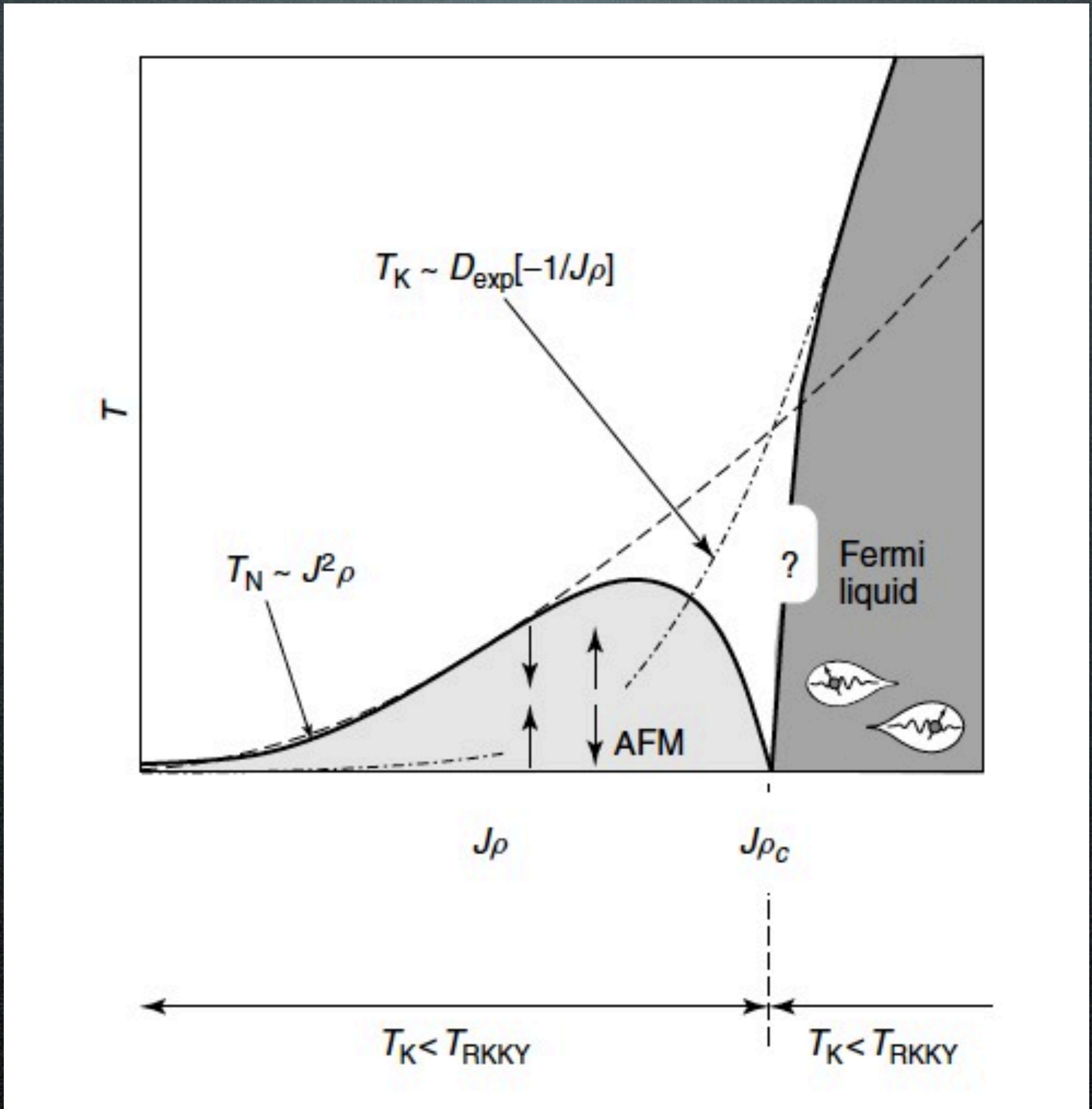
2. RKKY

$$H_{\mathrm{RKKY}} = \overbrace{-J^2\chi(\mathbf{x}-\mathbf{x}')}^{J_{\mathrm{RKKY}}(\mathbf{x}-\mathbf{x}')} \vec{S}(\mathbf{x}) \cdot \vec{S}(\mathbf{x}')$$

$$J_{\mathrm{RKKY}}(r) \sim -J^2\rho \frac{\cos 2k_{\mathrm{F}}r}{k_{\mathrm{F}}r}$$

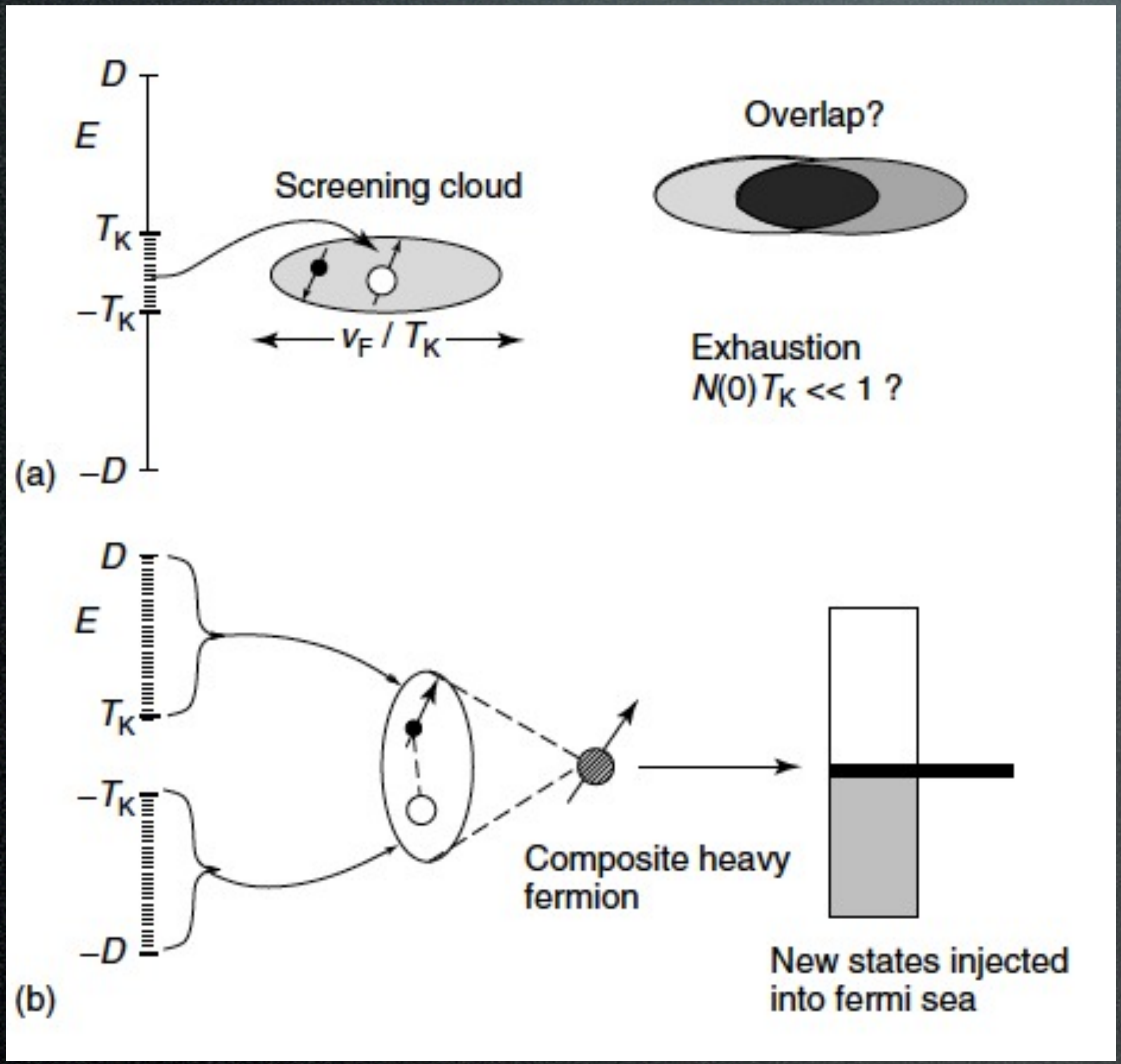
$$\langle \vec{\sigma}(r) \rangle \sim -J\rho \frac{\cos 2k_{\mathrm{F}}r}{|k_{\mathrm{F}}r|^3}$$

$$T_{\mathrm{RKKY}} = J^2\rho$$



$T_K \sim D_{exp}[-1/J\rho]$
$T_N \sim J^2\rho$
T
?
Fermi liquid
AFM
$J\rho$
$J\rho_c$
$T_K < T_{RKKY}$
$T_K < T_{RKKY}$

Relation between PAM and KLM



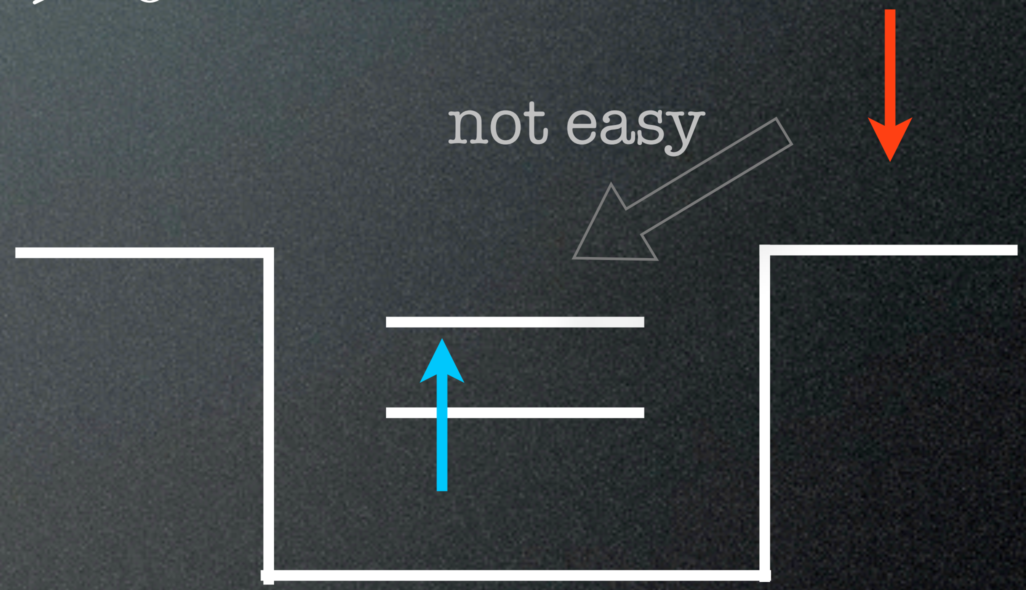
PAM -> KLM

KLM -> PAM

PAM -> KLM

$$H = \sum_{k\sigma} \epsilon_k n_{k\epsilon} + \sum_{k\sigma} V_k (c^\dagger_{k\sigma} f_\sigma + f^\dagger_\sigma c_{k_\sigma}) + \epsilon_f f^\dagger_\sigma f_\sigma + U f^\dagger_\uparrow f_\uparrow f^\dagger_\downarrow f_\downarrow$$

- Under the acquirement
  1. $\epsilon_f < 0, \quad \epsilon_f + U > 0$
  2. $\frac{\Delta}{|\epsilon_f|}, \frac{\Delta}{\epsilon_f + U} \ll 1$



$$\Delta = \pi V^2 \rho$$

PAM -> KLM

- Under the acquirement
  1. $\epsilon_f < 0, \quad \epsilon_f + U > 0$
  2. $\frac{\Delta}{|\epsilon_f|}, \frac{\Delta}{\epsilon_f + U} \ll 1$

$$H = \sum_{k\sigma} \epsilon_k n_{k\epsilon} + \sum_{k\sigma} V_k (c^\dagger_{k\sigma} f_\sigma + f^\dagger_\sigma c_{k_\sigma}) + \epsilon_f f^\dagger_\sigma f_\sigma + U f^\dagger_\uparrow f_\uparrow f^\dagger_\downarrow f_\downarrow$$

perform a Schrieffer-Wolff transformation

$$H_{PAM} \to \bar{H} = e^w H_{PAM} e^{-w}$$

$$H \simeq H_{spin-flip} + H_{scattering} + H_{quasispin-filp} + O(s^3)$$

$$H_{spin-flip} = -\frac{1}{2} J \sum S \cdot s$$

Effective Hamiltonian

- Despite that we have two approach to study the KI, they both can not really be solved without approximation..
- We wish to study the Hamiltonian in a form of

$$H_{eff} = \Phi^\dagger \begin{bmatrix} \tilde{\epsilon_c} & \tilde{V} \\ \tilde{V} & \tilde{\epsilon_f} \end{bmatrix} \Phi$$

$$\Phi^\dagger = (c^\dagger, f^\dagger)$$

PAM -> eff. H

- Employ a slave boson b_i , and run the mean field to eliminate $Uf^\dagger f f^\dagger f$

$$H = \sum_{k\sigma} \epsilon_k n_{k\epsilon} + \sum_{k\sigma} V_k (c^\dagger_{k\sigma} f_\sigma + f^\dagger_\sigma c_{k_\sigma}) + \epsilon_f f^\dagger_\sigma f_\sigma + U f^\dagger_\uparrow f_\uparrow f^\dagger_\downarrow f_\downarrow$$

$$H_{PAM} \to \sum_{k\sigma} \epsilon_k n_{k\epsilon} + \sum_{k\sigma} \tilde{V}_k (c^\dagger_{k\sigma} f_\sigma + f^\dagger_\sigma c_{k_\sigma}) + \tilde{\epsilon_f} f^\dagger_\sigma f_\sigma$$

$V \to \tilde{V} = bV$, $\epsilon_f \to b^2 \epsilon_f$ and b=$\langle \hat{b_i} \rangle$.

KLM -> eff. H

- By Hubbard Stratonovich transformation

$$\begin{aligned} H_{KLM} &= \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J \sum c^\dagger_{i\sigma} \mathbf{s}_{\sigma\sigma'} c_{i\sigma} \cdot \mathbf{S_i} \\ &\rightarrow \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum [f^\dagger_i W_i c_i + HC] + \frac{1}{J} Tr[W^\dagger_i W_i] \end{aligned}$$

Important Questions

Questions

- The competition between magnetic order and electronic quantum fluc.
- Coupling magnetic and electronic properties to develop new classes of material behaviors
- Some weird phenomena appear in specific Kondo insulators