Recap: U(1) slave-boson formulation of t-J model and mean field theory

• Mean field phase diagram

$$H = \sum_{\langle \mathbf{ij} \rangle} J\left(\mathbf{S}_{\mathbf{i}} \cdot \mathbf{S}_{\mathbf{j}} - \frac{1}{4}n_{\mathbf{i}}n_{\mathbf{j}}\right) - \sum_{\mathbf{ij}} t_{\mathbf{ij}}\left(c_{\mathbf{i}\sigma}^{\dagger}c_{\mathbf{j}\sigma} + \mathrm{H.c.}\right) \qquad c_{\mathbf{i}\sigma}^{\dagger} = \chi_{\mathbf{ij}} = \sum_{\sigma} \langle f_{\mathbf{i}\sigma}^{\dagger}f_{\mathbf{j}\sigma}\rangle \qquad \Delta_{\mathbf{ij}} = \langle f_{\mathbf{i}\uparrow}f_{\mathbf{j}\downarrow} - f_{\mathbf{i}\downarrow}f_{\mathbf{i}\uparrow}\rangle \qquad b = \langle b_{\mathbf{i}}\rangle$$
$$L_{1} = \sum_{i,\sigma} f_{\mathbf{i}\sigma}^{*}\left(\frac{\partial}{\partial\tau} - \mu_{F} + ia_{0}(\mathbf{r}_{\mathbf{i}})\right)f_{\mathbf{i}\sigma} + \sum_{i} b_{\mathbf{i}}^{*}\left(\frac{\partial}{\partial\tau} - \mu_{B} + ia_{0}(\mathbf{r}_{\mathbf{i}})\right)b_{\mathbf{i}} \qquad \text{approximation}$$

$$-\tilde{J}\chi\sum_{\langle \mathbf{ij}\rangle\sigma}\left(e^{ia_{\mathbf{ij}}}f_{\mathbf{i}\sigma}^{*}f_{\mathbf{j}\sigma}+\mathrm{h.c.}\right)-t\eta\sum_{\langle \mathbf{ij}\rangle}\left(e^{ia_{\mathbf{ij}}}b_{\mathbf{i}}^{*}b_{\mathbf{j}}+\mathrm{h.c.}\right)$$

Label	State	χ	Δ	b
Ι	Fermi liquid	$\neq 0$	= 0	$\neq 0$
II	Spin gap	$\neq 0$	$\neq 0$	= 0
III	<i>d</i> -wave	$\neq 0$	$\neq 0$	$\neq 0$
	superconducting			
IV	uRVB	$\neq 0$	= 0	= 0

$$c^{\dagger}_{\mathbf{i}\sigma} = f^{\dagger}_{\mathbf{i}\sigma}b_{\mathbf{i}}$$



SU(2) slave-boson formulation of t-J model and mean field theory

AR.

• Mean field phase diagram

$$\psi_{\mathbf{i}} = \begin{pmatrix} f_{\uparrow \mathbf{i}} \\ f_{\downarrow \mathbf{i}}^{\dagger} \end{pmatrix} \qquad h_{\mathbf{i}} = \begin{pmatrix} b_{1\mathbf{i}} \\ b_{2\mathbf{i}} \end{pmatrix} \qquad \qquad c_{\dagger \sigma}^{\dagger} = f_{\mathbf{i}\sigma}^{\dagger} \theta$$
$$c_{\uparrow \mathbf{i}} = \frac{1}{\sqrt{2}} h_{\mathbf{i}}^{\dagger} \psi_{\mathbf{i}} = \frac{1}{\sqrt{2}} \left(b_{1\mathbf{i}}^{\dagger} f_{\uparrow \mathbf{i}} + b_{2\mathbf{i}}^{\dagger} f_{\downarrow \mathbf{i}}^{\dagger} \right)$$
$$c_{\downarrow \mathbf{i}} = \frac{1}{\sqrt{2}} h_{\mathbf{i}}^{\dagger} \bar{\psi}_{\mathbf{i}} = \frac{1}{\sqrt{2}} \left(b_{1\mathbf{i}}^{\dagger} f_{\downarrow \mathbf{i}} - b_{2\mathbf{i}}^{\dagger} f_{\uparrow \mathbf{i}}^{\dagger} \right) \qquad \bar{\psi} = i\tau^{2} \psi^{*}$$

$$H_{\text{mean}} = \sum_{\langle \mathbf{ij} \rangle} \frac{3}{8} J \left[\frac{1}{2} \text{Tr}(U_{\mathbf{ij}}^{\dagger} U_{\mathbf{ij}}) + (\psi_i^{\dagger} U_{\mathbf{ij}} \psi_j + \text{h.c.}) \right] \\ - \frac{1}{2} \sum_{\langle \mathbf{ij} \rangle} t(h_{\mathbf{i}}^{\dagger} U_{\mathbf{ij}} h_{\mathbf{j}} + \text{h.c.}) - \mu \sum_{\mathbf{i}} h_{\mathbf{i}}^{\dagger} h_{\mathbf{i}} + \sum_{\mathbf{i}} a_0^l (\psi_{\mathbf{i}}^{\dagger} \tau^l \psi_{\mathbf{i}} + h_{\mathbf{i}}^{\dagger} \tau^l h_{\mathbf{i}}) \\ AB_{\mathbf{i}}$$







 $T_{\mathbf{X}}$

metal

doping

A. Aim and scope

- Discovery
 - Discovered in 2008 by Kamihara *et al.* in LaFeAsO with $T_c = 26$ K
 - Similarities with the cuprates
 - High- T_c
 - quasi-2D
 - Almost universal phase diagram



B. Fe-based superconductors

- Comparison with cuprates
 - Gap structure unclear after following experiments:
 - Penetration depth
 - ARPES
 - NMR
 - Phase sensitive Josephson tunneling
- Important questions
 - What is the gap structure?
 - What is pairing mechanism?
 - What is the role of disorder in experiments?



doping

- **B. Fe-based superconductors**
- Comparison with cuprates
 - Fe has weaker interactions
 - 2*p*-ligands (e.g. As) lie out of Fe plane
 - Multiple bands near the Fermi energy
 - Undoped or parent compounds are poor metals with SDW order
 - Coexistence of superconductivity and magnetism
 - No robust pseudogap region
 - Doping involves in-plane and out-ofplane substitution, e.g. Sr_{1-x}K_xFe₂As₂, Ba(Fe_{1-x}Co_x)₂As₂, etc.





- **B. Fe-based superconductors**
- Comparison with MgB₂
 - First example of multigap superconductivity
 - $T_c = 40 \text{ K} \Rightarrow$ higher than first cuprate!
 - Type-II multiband BCS superconductor
- Conceptual importance
 - Not an improvement from practical standpoint
 - T_c lower than cuprates (maximum 55 K)
 - Expensive to fabricate and difficult to work with
 - Pnictides demonstrated cuprate properties are **not** unique to high- T_c
 - Differences in cuprates and pnictides highlight critical ingredients of high- T_c



- **B.** Fe-based superconductors
- Families of pnictides/chalcogenides
 - 1111 family \rightarrow LaFeAsO \rightarrow LaFeAsO_{1-x} F_x
 - 122 family \rightarrow SrFe₂As₂
 - 111 family → LiFeAs
 - 11 family → FeTe



- **B. Fe-based superconductors**
- Gap symmetry and structure
 - Experiments ruled out triplet pairing
 - *s*-wave and *d*-wave have different symmetry
 - s_{++} and fully gapped s_{\pm} make sense in multiband picture
 - Gap symmetry \rightarrow phase change in **k**-space
 - Singlet vs. triplet → phase change in spinspace (no spin-orbit coupling)
 - Gap "structure" → everything else! e.g. phase change across Fermi sheets



II. Electronic structure

- **B.** Minimal band models
- Multiband nature
 - Unit cells \rightarrow "folded" (blue) and "unfolded" (green)
 - Treat height of Arsenic as a perturbation
 - Simplest model in "unfolded" zone

$$H = \sum_{\mathbf{k},\sigma,i=\alpha_{1},\alpha_{2},\beta_{1},\beta_{2}} \varepsilon_{\mathbf{k}}^{i} c_{i\mathbf{k}\sigma}^{\dagger} c_{i\mathbf{k}\sigma} \qquad \varepsilon_{\mathbf{k}}^{\alpha_{1,2}} = -\frac{\mathbf{k}^{2}}{2m_{1,2}} + \mu$$
$$\varepsilon_{\mathbf{k}}^{\beta_{1}} = \frac{(k_{x} - \pi/a)^{2}}{2m_{x}} + \frac{k_{y}^{2}}{2m_{y}} - \mu \qquad \varepsilon_{\mathbf{k}}^{\beta_{2}} = \frac{k_{x}^{2}}{2m_{y}} + \frac{(k_{y} - \pi/a)^{2}}{2m_{x}} - \mu$$

• Fermi sheets for electron-hole doped case (Fe²⁺ \rightarrow 3*d*⁶)





- A. Spin fluctuation pairing
- Historical: ferromagnetic spin fluctuations
 - Transition metals near ferromagnetism
 - Exchange of "paramagnons"



- A. Spin fluctuation pairing
- Antiferromagnetic spin fluctuations
 - Susceptibility strongly peaked near **Q**
 - Singlet interaction (always repulsive)

$$\Gamma_s(\mathbf{q}) = \frac{3}{2}U^2 \frac{\chi_0(\mathbf{q})}{1 - U\chi_0(\mathbf{q})}$$

• Self-consistent BCS gap equation

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k}'} \Gamma_s(\mathbf{k}, \mathbf{k}') \frac{\Delta'_{\mathbf{k}}}{2E'_{\mathbf{k}}} \tanh\left(\frac{E'_{\mathbf{k}}}{2T}\right)$$

• Solution possible for

$$\Delta_{\mathbf{k}} = -\Delta_{\mathbf{k}+\mathbf{Q}}$$

• In cuprates, χ is peaked at $\mathbf{Q} = (\pi, \pi)$

$$\Delta_{\mathbf{k}}^{d,s} = \Delta_0(\cos(k_x a) \mp \cos(k_y a))$$

• In pnictides, χ is peaked at $\mathbf{Q} = (\pi, 0) \rightarrow s_{\pm}$ pairing





- A. Spin fluctuation pairing
- Spin fluctuation pairing in multi-orbital systems



- Effective pair scattering vertex between bands *i* and *j* in the singlet channel $\Gamma_{ij}(\mathbf{k}, \mathbf{k}') = \operatorname{Re} \left[\sum_{\ell_1 \ell_2 \ell_3 \ell_4} a_{\nu_i}^{\ell_2, *}(\mathbf{k}) a_{\nu_i}^{\ell_3, *}(-\mathbf{k}) \Gamma_{\ell_1 \ell_2 \ell_3 \ell_4}(\mathbf{k}, \mathbf{k}', \omega = 0) a_{\nu_j}^{\ell_1}(\mathbf{k}') a_{\nu_j}^{\ell_4}(-\mathbf{k}') \right]$ $a_{\nu}^{\ell}(\mathbf{k}) = \langle \ell | \nu \mathbf{k} \rangle \quad \mathbf{k} \in C_i \quad \mathbf{k}' \in C_j$ $l_1^{\dagger} \quad l_2^{\dagger}$
- Orbital vertex functions Spin fluctuations $\Gamma_{\ell_1\ell_2\ell_3\ell_4}(\mathbf{k},\mathbf{k}',\omega) = \left[\frac{3}{2}\bar{U}^s\chi_1^{\text{RPA}}(\mathbf{k}-\mathbf{k}',\omega)\bar{U}^s + \frac{1}{2}\bar{U}^s - \frac{1}{2}\bar{U}^c\chi_0^{\text{RPA}}(\mathbf{k}-\mathbf{k}',\omega)\bar{U}^c + \frac{1}{2}\bar{U}^c\right]_{\ell_1\ell_2\ell_3\ell_4}$

Charge/orbital fluctuations

$$\begin{array}{c} 1 \\ \Gamma_{1111} \\ \Gamma_{1111} \\ \Gamma_{1111} \\ \Gamma_{1111} \\ \Gamma_{1121} \\ \Gamma_{1122} \\$$

 $\Gamma_{l_1 l_2 l_3 l_4}$

 l_{4}^{\downarrow}

n = 5.95

d_{xz} -

 $d_{yz} - d_{xy}$

III. Theoretical background A. Spin fluctuation pairing

- Results of microscopic theory
 - Gap symmetry and structure

 $\Delta({\bf k}) = \Delta g({\bf k})$

• Orbital vertex functions



$$\lambda[g(\mathbf{k})] = -\frac{\sum_{ij} \oint_{C_i} \frac{d\mathbf{k}_{\parallel}}{v_F(\mathbf{k})} \oint_{C_j} \frac{d\mathbf{k}'_{\parallel}}{v_F(\mathbf{k}')} g(\mathbf{k}) \Gamma_{ij}(\mathbf{k}, \mathbf{k}') g(\mathbf{k}')}{(2\pi)^2 \sum_i \oint_{C_i} \frac{d\mathbf{k}_{\parallel}}{v_F(\mathbf{k})} g^2(\mathbf{k})} \qquad \mathbf{k} \in C_i \qquad \mathbf{k}' \in C_j$$
$$v_{F,\nu}(\mathbf{k}) = |\nabla_{\mathbf{k}} E_{\nu}(\mathbf{k})|$$



- A. Spin fluctuation pairing
- Physical origins of anisotropy of pair state and node formation
 - Intra-orbital pairing between α and β Fermi sheets \rightarrow favors s_{\pm}
 - Sub-leading inter-orbital between β₁ and β₂
 Fermi sheets → favors nodes → frustrates
 s_±
 - Hole doping $(n < 6) \rightarrow$ appearance of γ pocket
 - Appearance of γ pocket $\Rightarrow \beta_1 \beta_2$ scattering causes weaker s_{\pm} frustration
 - Height of Arsenic above Fe-plane → appearance of γ pocket and isotropy of s_± state in 1111 family

n = 5.95 $\pi \begin{bmatrix} d_{xz} & d_{yz} & d_{xy} \\ \beta_2 & \gamma \end{bmatrix}$ $\pi \begin{bmatrix} \beta_2 & \gamma \\ 0 \end{bmatrix}$ $\pi \begin{bmatrix} \beta_2 & \gamma \\ 0 \end{bmatrix}$ $\pi \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \end{bmatrix}$

 $\overline{\mathbf{x}}$

- **B.** Alternative approaches
- Excitonic Superconductivity
 - Proposed by Little and Ginzburg
 - Exciton + phonon → CDW → suppress superconductivity
- Orbital fluctuations
 - Ordering of Fe 3d orbitals possible
 - Recall Hamiltonian

$$H = H_0 + \overline{U} \sum_{i,\ell} n_{i\ell\uparrow} n_{i\ell\downarrow} + \overline{U}' \sum_{i,\ell'<\ell} n_{i\ell} n_{i\ell'} + \overline{J} \sum_{i,\ell'<\ell} \sum_{\sigma,\sigma'} c^{\dagger}_{i\ell\sigma} c^{\dagger}_{i\ell'\sigma'} c_{i\ell\sigma'} c_{i\ell'\sigma} + \overline{J}' \sum_{i,\ell'\neq\ell} c^{\dagger}_{i\ell\uparrow} c^{\dagger}_{i\ell\downarrow} c_{i\ell'\downarrow} c_{i\ell'\uparrow}$$

• Cannot have $\bar{U}' > \bar{U}, \ \bar{J}' > \bar{J}$ for purely electronic interactions



phonon

Orbital

electron

fluctuation

A. Does the gap in FeBS change sign?

• Spin-resonance peak



A. Does the gap in FeBS change sign?

- Josephson junctions
 - *d*-wave symmetry of cuprates confirmed by Josephson effect
 - Phase shift of π between orthogonal planes
 - Unfortunately pnictides don't have spatial anisotropy
 - Phase difference across electron and hole Fermi sheets for s[±] pairing
 - Solution → Measure Josephson effect across epitaxially grown interface between electron- and holedoped pnictide





A. Does the gap in FeBS change sign?

• Quasiparticle interference $\propto u_{\mathbf{k}}u_{\mathbf{k}'} + v_{\mathbf{k}}v_{\mathbf{k}'}$

$$|v_{\mathbf{k}}|^2 = 1 - |u_{\mathbf{k}}|^2 = \frac{1}{2} \left(1 - \frac{\epsilon_{\mathbf{k}}}{E_{\mathbf{k}}} \right)^{1/2} \quad E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta_{\mathbf{k}}^2}$$

• QPI in the Cuprates

$$\epsilon_{\mathbf{k}} = -2\chi_0 \left(\cos(k_x) + \cos(k_y)\right)$$

$$\Delta_{\mathbf{k}} = 2\Delta_0 \left(\cos(k_x) - \cos(k_y) \right)$$

- QPI in the Pnictides → no "hot spots"
- Bragg vs. QPI?







1.0 mV

q₂

10 T

Inc.

A. Does the gap in FeBS change sign?

- Coexistence of magnetism and superconductivity
 - Co-doped BaFe₂As₂ → Microscopic coexistence of weak antiferromagnetism and superconductivity
 - s_{\pm} or s_{++} states coexists with SDW
 - In-plane thermal conductivity experiment
 → no *c*-axis nodal lines
 - s₊₊ pairing → SDW-induced BZ band folding → c-axis line nodes → s₊₊ pairing cannot coexist with SDW



B. Evidence for low-energy subgap excitations

- Evidence for very low energy excitations consistent with gap nodes
- Bulk probes provide consistent picture of the evolution of the low-energy quasiparticle density across the phase diagram



- **B. Evidence for low-energy subgap excitations**
- Penetration depth
 - $\Delta\lambda \propto \exp(-\Delta_{\min}/T) \rightarrow \text{fully gapped}$ (optimally doped $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$)
 - $\Delta \lambda \propto T \rightarrow$ line nodes (1111 family)
 - $\Delta\lambda \propto T^2 \rightarrow$ disorder (122 family)





- **B. Evidence for low-energy subgap excitations**
- Specific heat
 - Nodes \rightarrow Volovik effect $\rightarrow C/T \propto H^{1/2}$
 - Fully gapped \rightarrow vortex cores states $\rightarrow C/T \propto H$
 - Fe(Te,Se) specific heat oscillations



B. Evidence for low-energy subgap excitations

- The ARPES "paradox"
 - The most direct probe of gap structure \rightarrow proved *d*-wave pairing in cuprates

 S_+

nodal s_+

- No ARPES study has seen nodes in the gap
- Possible explanations of the paradox
 - Surface electronic reconstruction
 - Surface DFT on BaFe₂As₂ \rightarrow additional d_{xy} pocket
 - d_{xy} pocket stabilizes isotropic pair state
 - Surface depairing
 - Surface roughness → in-plane intraband scattering → destroys gap anisotropy
 - Resolution issues

Summary

- Bardeen-Cooper Schrieffer (conventional) superconductors
 - Discovered in **1911** by Kamerlingh-Onnes
 - Fully gapped Bogoliubov quasiparticle spectrum
 - Important effects
 - Vanishing resistivity
 - Meissner effect (London penetration depth)
 - Coherence effects (coherence length)
- Heavy-fermion superconductors
 - Discovered by Steglich *et al*. in **1979**
 - Key ingredients
 - Lattice of *f*-electrons
 - Conduction electrons
 - Multiple superconducting phases





Summary

- Electronic structure
 - Relevant physics confined to 2D
 - The "t-J" model

$$H = P\left[-\sum_{\langle \mathbf{ij}\rangle,\sigma} t_{\mathbf{ij}} c_{\mathbf{i}\sigma}^{\dagger} c_{\mathbf{i}\sigma} + J \sum_{\langle \mathbf{ij}\rangle} \left(\mathbf{S}_{\mathbf{i}} \cdot \mathbf{S}_{\mathbf{j}} - \frac{1}{2} n_{\mathbf{i}} n_{\mathbf{j}}\right)\right] P$$

- Universal phase diagram
- Phenomenology of the cuprates
 - Experimental signatures of the pseudogap phase

 $E(k_x, k_y)$

-3

-3

 k_x

3

- Nodal quasiparticles
- Slave bosons
 - Slave fermions and bosons
 - U(1) & SU(2) gauge theory



 $k_x(\pi/a)$

Summary

- Iron-based (pnictide) superconductors
 - Discovered in 2008 by Kamihara
 - Physics confined to 2D like cuprates
 - Pseudogap replaced by "nematic phase"

• Gap structure

- Theory and some experiments \rightarrow s±
- Contradictory experimental evidence
 - ARPES
 - Specific heat
 - Penetration depth





Thanks for listening!



Future Topics

- Heavy fermion materials
 - Not limited to superconductivity
 - e.g. Kondo topological insulators
- Exotic topics motivated by the cuprates
 - Gauge theories and confinement physics
 - Quantum critical point (QCP)
- Competing interpretations of cuprate high-Tc?
 - Stripe or no stripe?
 - Is slave boson picture nonsense?



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How to detect fluctuating stripes in the high-temperature superconductors

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This article discusses fluctuating order in a quantum disordered phase proximate to a quantum critical point, with particular emphasis on fluctuating stripe order. Optimal strategies are derived for extracting information concerning such local order from experiments, with emphasis on neutron scattering and scanning tunneling microscopy. These ideas are tested by application to two model systems—an exactly solvable one-dimensional (1D) electron gas with an impurity, and a weakly interacting 2D electron gas. Experiments on the cuprate high-temperature superconductors which can be analyzed using these strategies are extensively reviewed. The authors adduce evidence that stripe correlations are widespread in the cuprates. They compare and contrast the advantages of two limiting perspectives on the high-temperature superconductor: weak coupling, in which correlation effects are treated as a perturbation on an underlying metallic (although renormalized) Fermi-liquid state, and strong coupling, in which the magnetism is associated with well-defined localized spins, and stripes are viewed as a form of micro phase separation. The authors present quantitative indicators that the latter view better accounts for the observed stripe phenomena in the cuprates.