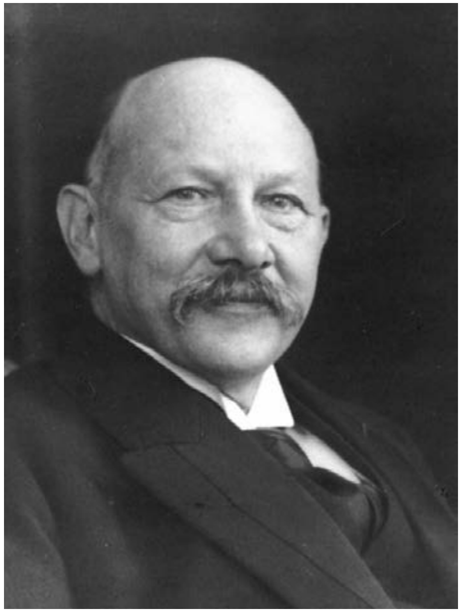
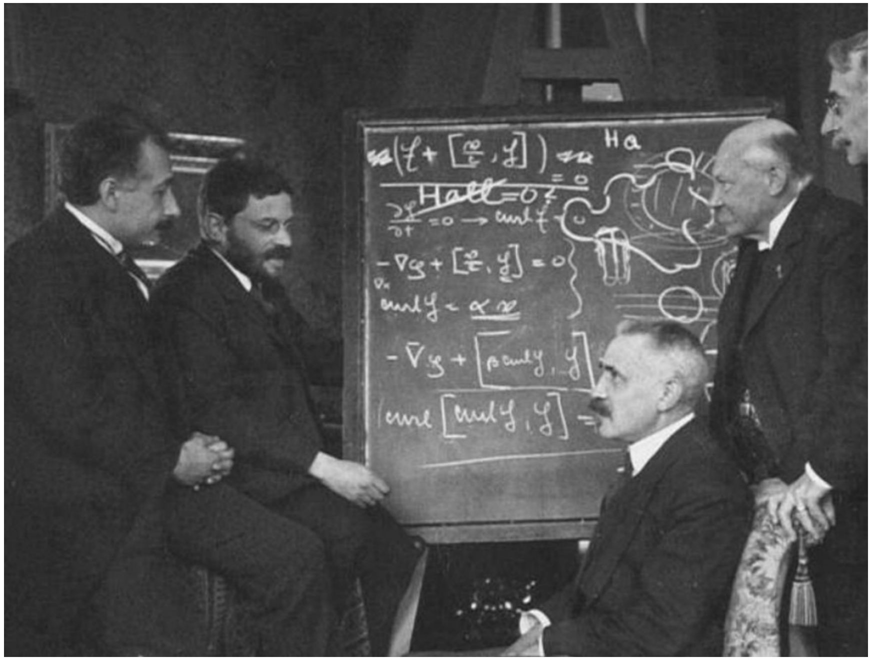
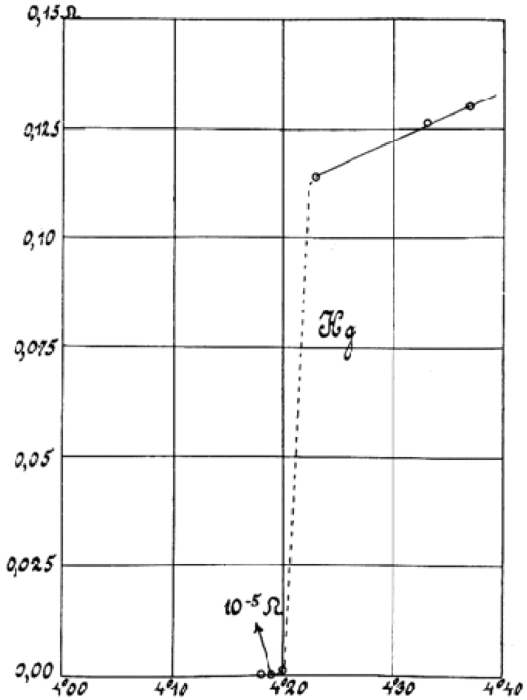
History of superconductivity

- **Liquefaction of ^{4}He**
  - Heike Kamerlingh Onnes produces liquid ^{4}He on 10 July, 1908
  - On 8 April, 1911 he discovered superconduct-ivity in a solid Hg wire at 4.2 K
  - Quantum origins of superconductivity a mystery until 1957

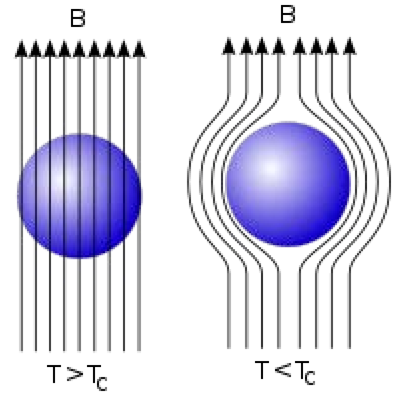




Einstein, Ehrenfest, Langevin, Kamerlingh Onnes, and Weiss at a workshop in Leiden October 1920. The blackboard discussion, on the Hall effect in superconductors



Phenomenology of superconductivity



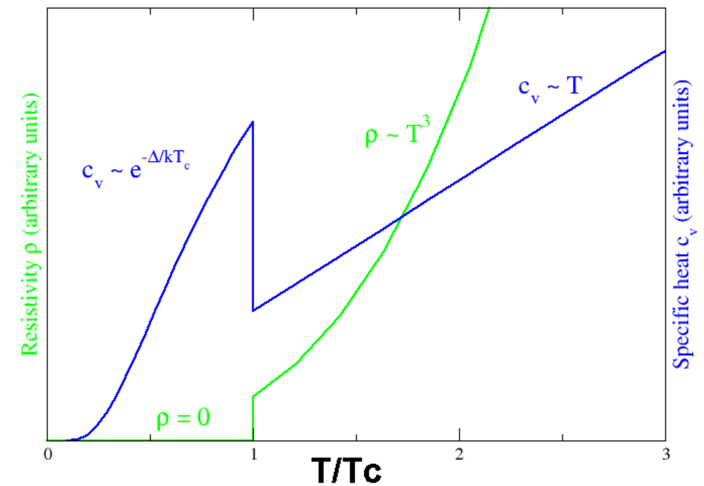
- **Experimental facts**
  - Vanishing resistivity, at $T = 0$, up to $\omega = 3.5 k_B T_C/\hbar$
  - Zero resistivity, at $T > 0$, **only** at $\omega = 0$
  - Meissner effect (1933) ➔ expulsion of magnetic field in the bulk
  - Jump in specific heat ≈ 3 times γT_C
  - Isotope effect

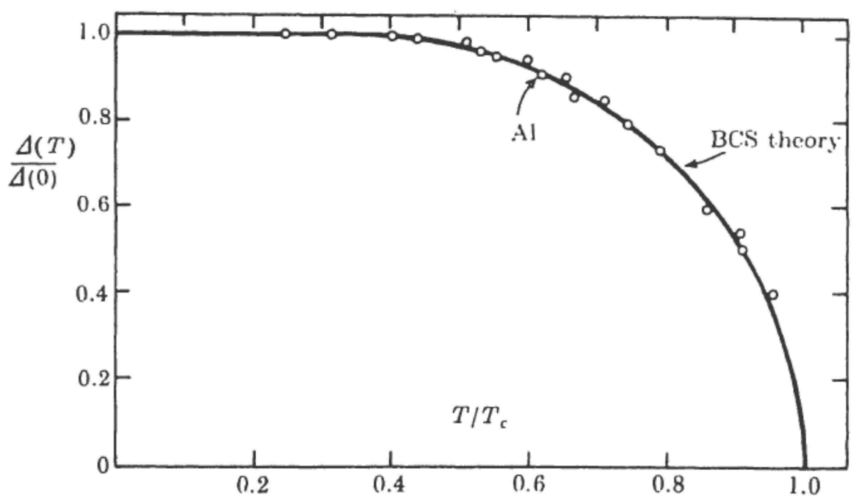
$$T_c \sim \frac{1}{M^\alpha} \sim H_0 \quad \left(\alpha \approx \frac{1}{2}\right)$$

  - Energy gap $\Delta(T)$

$$\Delta(T) = 3.5\, k_B T_c \sqrt{1 - \frac{T}{T_c}}$$

  - Coherence effects





Phenomenology of superconductivity

- **Phenomenological theories**
  - Gorter-Casimir Model (1934) ➔ "artificial" two-fluid model

$$F(x,T) = x^{1/2} f_n(T) + (1-x) f_s(T)$$

"Superfluid" free energy

"Normal" fluid free energy

$$\left[\frac{dF(x,T)}{dx}\right]_{x=x^*} = 0 \qquad x^* = \left(\frac{T}{T_c}\right)^4$$

$$f_n(T) = -\frac{1}{2}\gamma T^2 \qquad f_s(T) = -\beta = -\frac{1}{4}\gamma T_c^2$$

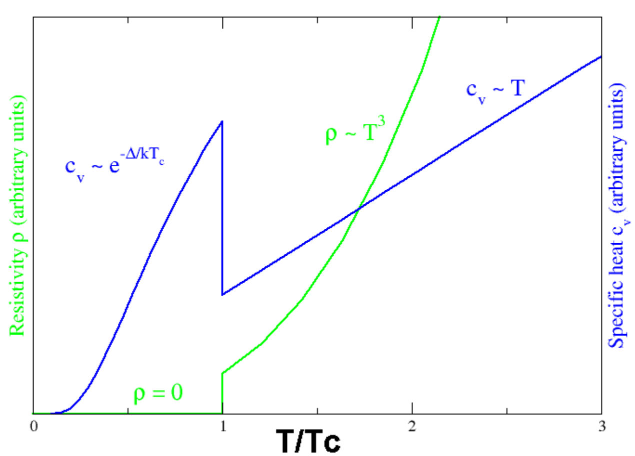
$$F(T) \equiv F(x^*,T) = -\beta\left[1 - \left(\frac{T}{T_c}\right)^4\right] \Rightarrow C_{es}(T) = -T\frac{\partial^2 F(T)}{\partial T^2} = 3\gamma T_c \left(\frac{T}{T_c}\right)^3$$

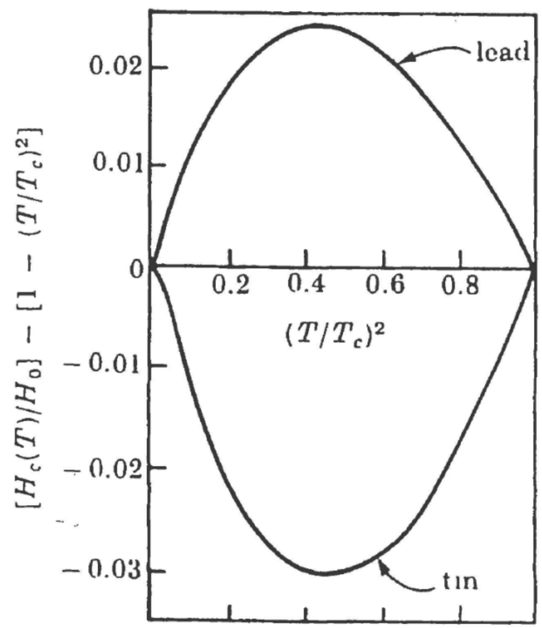
  - Temperature-dependent critical magnetic field

$$\frac{H_c^2(T)}{8\pi} = F_n(T) - F_s(T)$$

$$\Downarrow$$

$$H_c(T) = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$





Phenomenology of superconductivity

- **Phenomenological theories**
  - The London Theory (1935) ➔ realistic two-fluid model
  - Equations of motion of the two-fluid model

$$\frac{d\mathbf{J}_s}{dt} = \frac{n_s e^2}{m}\mathbf{E} \quad (\mathbf{J}_s = -e n_s \mathbf{v}_s)$$

$$\mathbf{J}_n = \sigma_n \mathbf{E} \quad (\mathbf{J}_n = -e n_n \mathbf{v}_n)$$

  - Londons' second equation

$$\nabla \times \mathbf{J}_s = -\frac{n_s e^2}{mc}\mathbf{B}$$

$$\Downarrow$$

$$\nabla \times \nabla \times \mathbf{B} = \frac{4\pi}{c}\nabla \times \mathbf{J}_s$$

$$\nabla^2 \mathbf{B} = \frac{4\pi n_s e^2}{mc^2}\mathbf{B} = \frac{1}{\lambda_L^2}\mathbf{B} \Longrightarrow \boxed{\lambda_L = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}}$$

$$\Uparrow$$

$$B(x) = B(0)e^{-x/\lambda_L}$$

  - Gorter-Casimir + Londons theory

$$(1-x) = 1 - \left(\frac{T}{T_c}\right)^4 = \frac{n_s(T)}{n} \Longrightarrow \lambda(T) = \frac{\lambda(0)}{\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{1/2}}$$

Bardeen-Cooper-Schrieffer (BCS) Theory

- **Pairing hypothesis**
  - Hubbard model with attractive interaction

$$H_{\mathrm{BCS}} = \sum_{\mathbf{k}\sigma} \varepsilon(\mathbf{k}) c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow} c_{-\mathbf{k}',\downarrow} c_{\mathbf{k}',\uparrow}$$

  - Composite "boson"

$$\Delta(\mathbf{k}) = \sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \langle c_{-\mathbf{k}',\downarrow} c_{\mathbf{k}',\uparrow} \rangle \qquad \Delta^*(\mathbf{k}) = \sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \left\langle c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow} \right\rangle$$

  - Assuming mean-field works

$$H^{\mathrm{mean}}_{\mathrm{BCS}} = \sum_{\mathbf{k}\sigma} \varepsilon(\mathbf{k}) c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \sum_{\mathbf{k}} \Delta^*(\mathbf{k}) c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow} + \sum_{\mathbf{k}} \Delta(\mathbf{k}) c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow}$$

$$H^{\mathrm{mean}}_{\mathrm{BCS}} = \sum_{\mathbf{k}} \begin{bmatrix} c_{\mathbf{k},\uparrow} & c^{\dagger}_{-\mathbf{k},\downarrow} \end{bmatrix} \begin{bmatrix} \varepsilon(\mathbf{k}) & \Delta^*(\mathbf{k}) \\ \Delta(\mathbf{k}) & -\varepsilon(\mathbf{k}) \end{bmatrix} \begin{bmatrix} c^{\dagger}_{\mathbf{k},\uparrow} \\ c_{-\mathbf{k},\downarrow} \end{bmatrix}$$

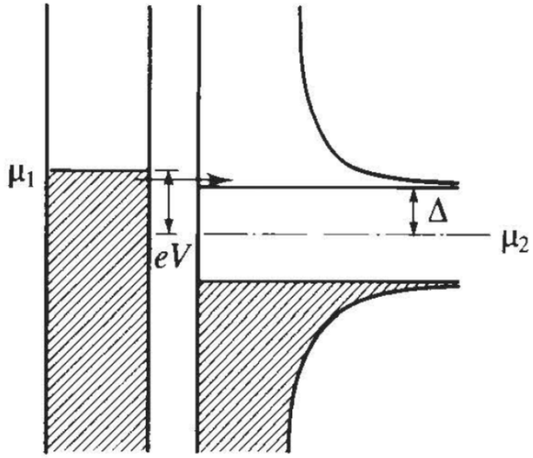
  - Find eigenvalues and eigenvectors!

$$E_{\pm}(\mathbf{k}) = \pm\sqrt{\varepsilon^2(\mathbf{k}) + |\Delta(\mathbf{k})|^2}$$

Bogoliubov quasiparticles

$$\gamma_{\mathbf{k},\uparrow} = u_{\mathbf{k}} c_{\mathbf{k},\uparrow} - v_{\mathbf{k}} c^{\dagger}_{-\mathbf{k},\downarrow} \quad |u_{\mathbf{k}}|^2 = \frac{1}{2}\left(1 + \frac{\varepsilon(\mathbf{k})}{E(\mathbf{k})}\right)$$

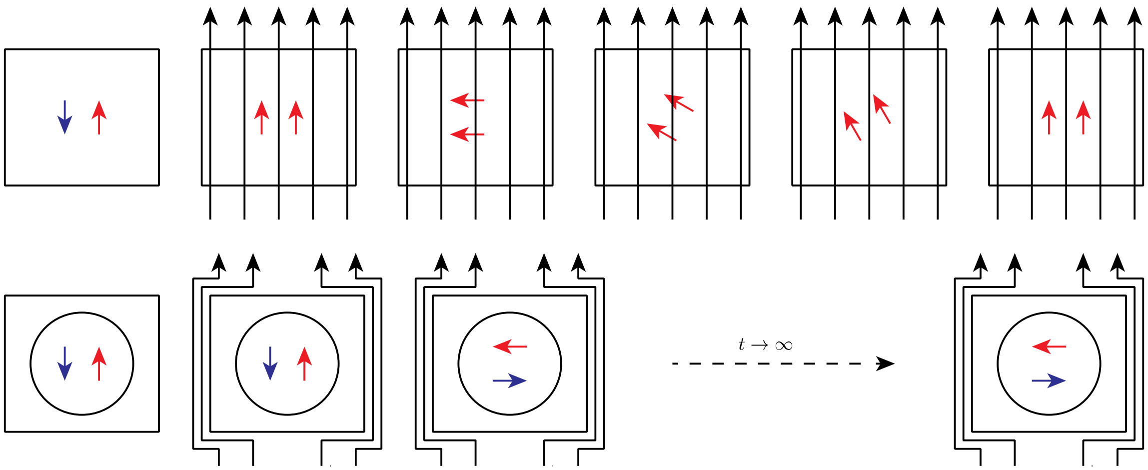
$$\gamma_{\mathbf{k},\downarrow} = u_{\mathbf{k}} c_{\mathbf{k},\downarrow} + v_{\mathbf{k}} c^{\dagger}_{-\mathbf{k},\uparrow} \quad |v_{\mathbf{k}}|^2 = \frac{1}{2}\left(1 - \frac{\varepsilon(\mathbf{k})}{E(\mathbf{k})}\right)$$

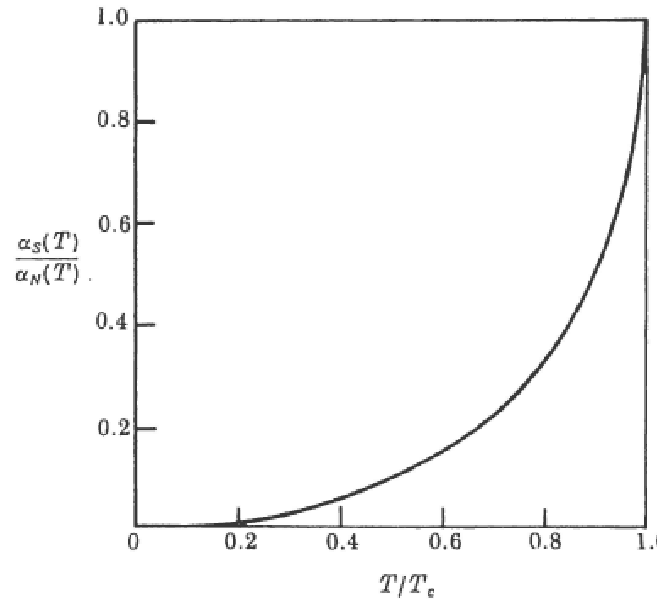


Experimental success of BCS

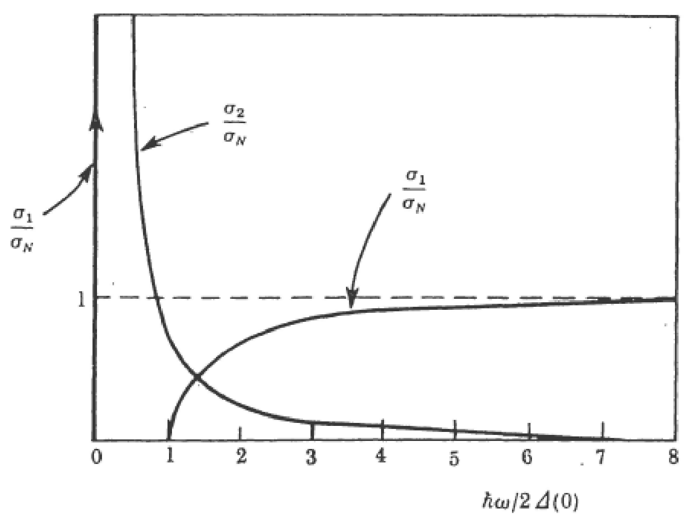
$$\frac{\alpha_S(T)}{\alpha_N(T)} = \frac{2}{\exp\left[\frac{\Delta(T)}{k_B T}\right] + 1}$$

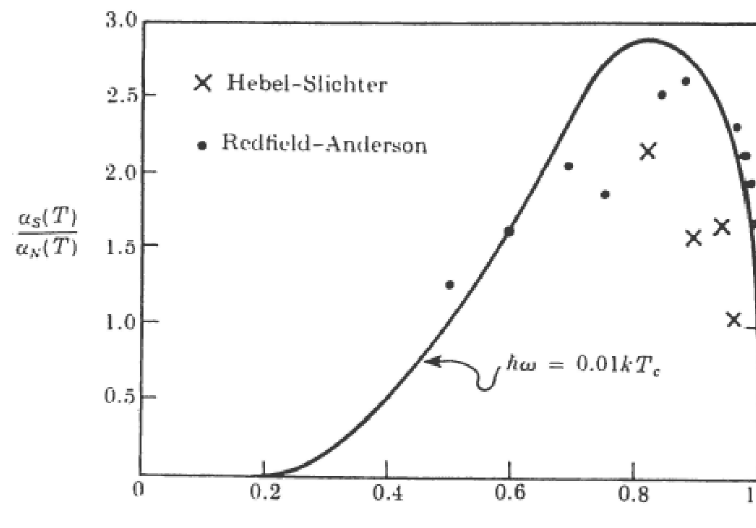
- **Acoustic attenuation rate**
- **Nuclear magnetic Resonance (NMR)**





- **Electromagnetic absorption**



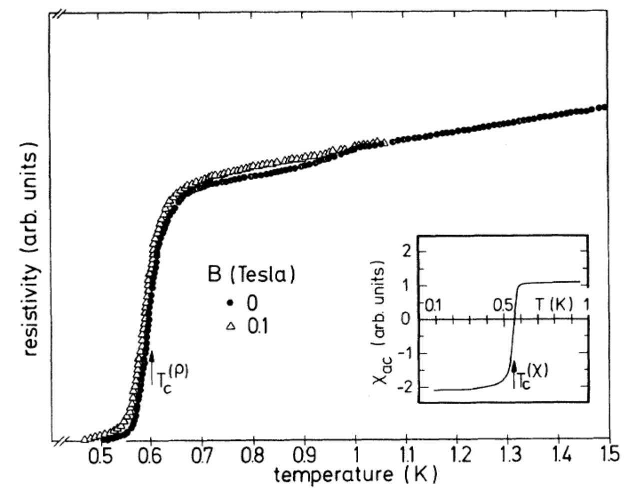


Part II: Heavy-fermion Superconductivity

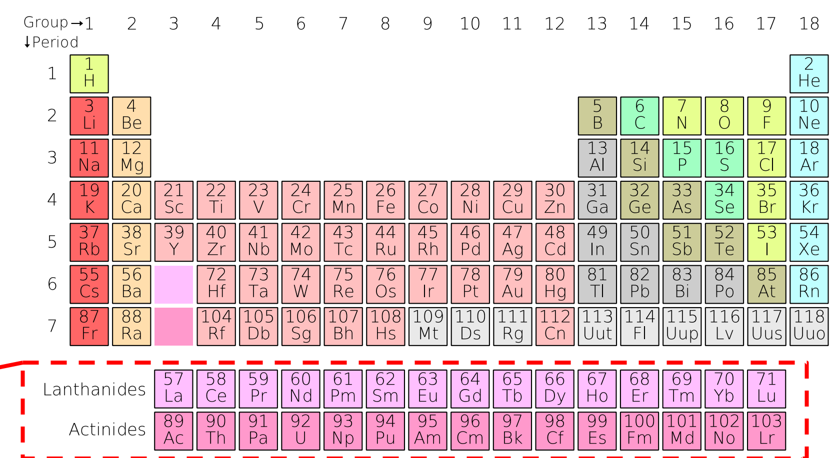
- **History**
- **Local moments and the Kondo lattice**
    - The Kondo effect
    - Poor man's RG
    - Kondo lattice
    - Heavy-fermion behavior
- **Heavy-fermion superconductivity**
    - Phenomenology
    - "BCS-like" theory
    - Superconducting phases of UPt_3
    - The FFLO phase

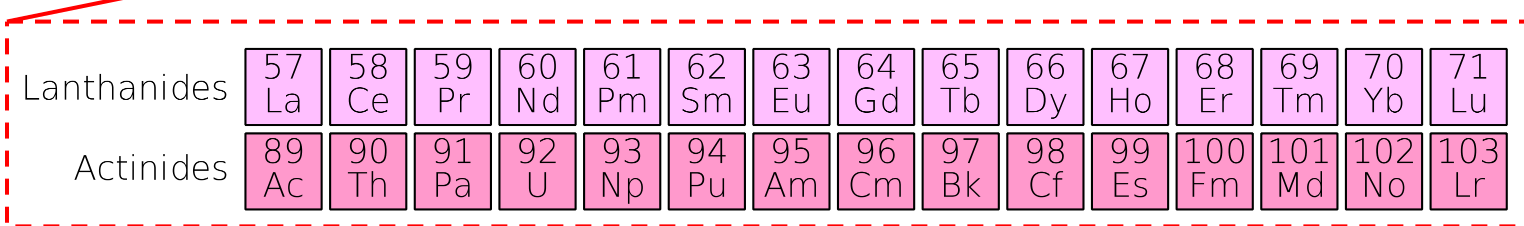
Introduction to “Heavy-fermion” systems

- **History**
  - Steglich *et al.* discovered super-conductivity in $CeCu_2Si_2$ with $T_c \approx$ 0.5 K
  - Postulated Cooper pairing of “heavy quasiparticles”



- **Heavy-fermion systems**
  - Ingredient 1: **lattice** of *f*-electrons
  - Ingredient 2: conduction electrons





Local moments and the Kondo lattice

- **The Anderson Model**
  - Localized *f*-electrons hybridize with the itinerant electrons

$$H = \overbrace{\sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} n_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\sigma} V(\mathbf{k}) \left[c^{\dagger}_{\mathbf{k}\sigma} f_{\sigma} + f^{\dagger}_{\sigma} c_{\mathbf{k}\sigma} \right]}^{H_{\text{resonance}}} + \underbrace{E_f n_f + U n_{f\uparrow} n_{f\downarrow}}_{H_{\text{atomic}}}$$

  - Quantum states in the atomic limit

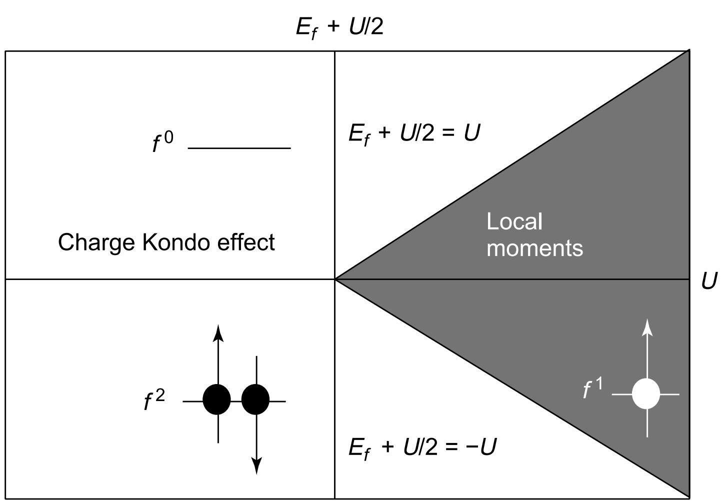
$$\left.\begin{array}{ll} |f^2\rangle & E(f^2) = 2E_f + U \\ |f^0\rangle & E(f^0) = 0 \end{array}\right\} \quad \text{non-magnetic}$$

$$|f^1 \uparrow\rangle, \quad |f^1 \downarrow\rangle \qquad E(f^1) = E_f. \qquad \text{magnetic.}$$

  - Cost of inducing a "valence fluctuation" by removing or adding an electron to the f^1 state is positive

$$E(f^0) - E(f^1) = -E_f > 0 \Rightarrow U/2 > E_f + U/2$$

$$E(f^2) - E(f^1) = E_d + U > 0 \Rightarrow E_d + U/2 > -U/2$$



Local moments and the Kondo lattice

- **The Anderson Model**
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$$H = \overbrace{\sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} n_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\sigma} V(\mathbf{k}) \left[c^{\dagger}_{\mathbf{k}\sigma} f_{\sigma} + f^{\dagger}_{\sigma} c_{\mathbf{k}\sigma} \right]}^{H_{\text{resonance}}} + \underbrace{E_f n_f + U n_{f\uparrow} n_{f\downarrow}}_{H_{\text{atomic}}}$$

  - Virtual bound-state formation resonance linewidth

$$\Delta = \pi \sum_{\mathbf{k}} |V(\mathbf{k})|^2 \delta(\epsilon_{\mathbf{k}} - \mu) = \pi V^2 \rho \qquad V(\mathbf{k}) = \langle \mathbf{k} | V_{\text{atomic}} | f \rangle$$

  - Two approaches to the Anderson model
    - "Atomic" picture ➔ tune hybridization strength
    - "Adiabatic" picture ➔ tune interaction strength
  - Resolution of discrepancy between two pictures ➔ "tunneling" of local moment between spin "up" and "down"

$$e^{-}_{\downarrow} + f^{1}_{\uparrow} \rightleftharpoons e^{-}_{\uparrow} + f^{1}_{\downarrow}$$

Local moments and the Kondo lattice

- **Adiabaticity and the Kondo resonance**
  - *f*-electron spectral function

$$A_f(\omega) = \frac{1}{\pi}\mathrm{Im}\left[G_f(\omega - i\delta)\right] \qquad G_f(\omega) = -i\int_{-\infty}^{\infty} dt \langle \mathrm{T} f_\sigma(t) f^\dagger_\sigma(0)\rangle e^{i\omega t}$$

$$A_f(\omega) = \begin{cases} \overbrace{\sum_\lambda \left|\langle\lambda|f^\dagger_\sigma|\phi_0\rangle\right|^2 \delta(\omega - [E_\lambda - E_0]),}^{\text{Energy distribution of state formed by adding one } f \text{ electron}} & (\omega > 0) \\ \underbrace{\sum_\lambda \left|\langle\lambda|f_\sigma|\phi_0\rangle\right|^2 \delta(\omega - [E_0 - E_\lambda]),}_{\text{Energy distribution of state formed by removing an } f \text{ electron}} & (\omega < 0) \end{cases}$$

  - *f*-charge on the ion

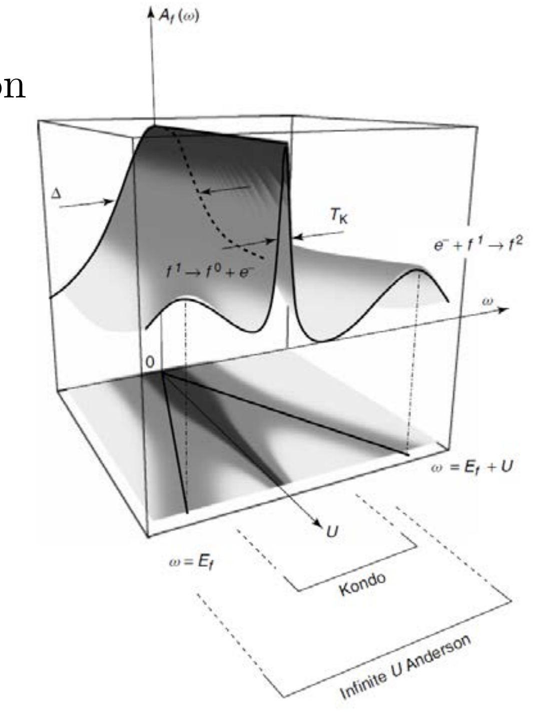
$$\langle n_f \rangle = 2\int_{-\infty}^{0} d\omega\, A_f(\omega)$$

  - Phase and amplitude of spectral function at $\omega = 0$

$$A_f(\omega = 0) = \frac{\sin^2(\delta_f)}{\pi\Delta}$$

  - Friedel sum rule

$$\sum_\sigma \frac{\delta_{f\sigma}}{\pi} = 2\frac{\delta_f}{\pi} = n_f$$

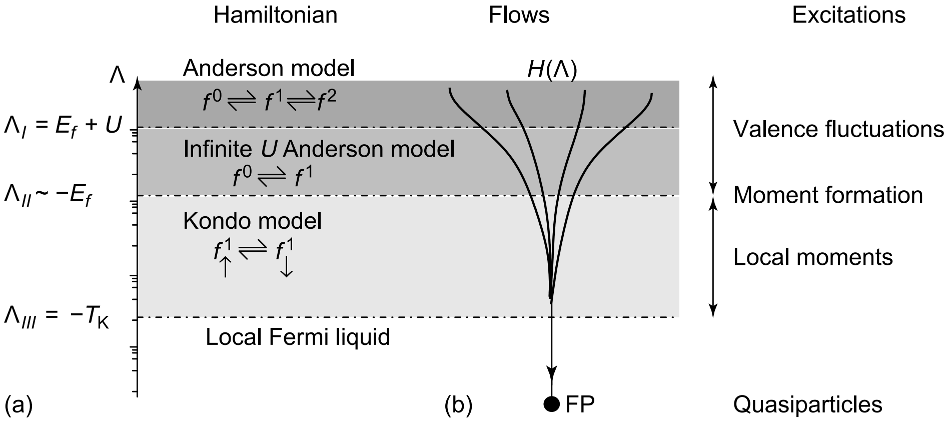


Local moments and the Kondo lattice

- **Hierarchies of energy scales**
  - Renormalization concept

$$H = \left[\begin{array}{c|c} \frac{H_L}{V} & \frac{V^\dagger}{H_H} \end{array} \right]$$

$$H(\Lambda) \rightarrow U H(\Lambda) U^\dagger = \left[\begin{array}{c|c} \frac{\tilde{H}_L}{0} & \frac{0}{\tilde{H}_H} \end{array} \right]$$



  - Renormalized Hamiltonian

$$H(\Lambda') = \tilde{H}_L = H_L + \delta H$$

  - Infinite-*U* Anderson model

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} n_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\sigma} V(\mathbf{k}) \left[c^\dagger_{\mathbf{k}\sigma} X_{0\sigma} + X_{\sigma 0} c_{\mathbf{k}\sigma} \right] + E_f \sum_{\sigma} X_{\sigma\sigma}$$

  - "Hubbard operators"

$$X_{\sigma\sigma} = |f^1 : \sigma\rangle\langle f^1 : \sigma| \qquad X_{0\sigma} = |f^0\rangle\langle f^1 \sigma| \qquad X_{\sigma 0} = |f^1 : \sigma\rangle\langle f^0|$$

  - For the symmetric Anderson model we have: $\Lambda_{\mathrm{I}} = \Lambda_{\mathrm{II}}$

Local moments and the Kondo lattice

- **Hierarchies of energy scales**
  - Schrieffer Wolff transformation

$$e^-_\uparrow + f^1_\downarrow \leftrightarrow f^2 \leftrightarrow e^-_\downarrow + f^1_\uparrow \qquad \Delta E_{\mathrm{I}} \sim U + E_f$$

$$h^+_\uparrow + f^1_\downarrow \leftrightarrow f^0 \leftrightarrow h^+_\downarrow + f^1_\uparrow \qquad \Delta E_{\mathrm{II}} \sim -E_f$$

  - Energy of singlet lowered by –2*J*

$$J = V^2 \left[\frac{1}{\Delta E_{\mathrm{I}}} + \frac{1}{\Delta E_{\mathrm{II}}} \right]$$

  - Matrix elements associated with valence fluctuations

$$|f^1 c^1\rangle = \frac{1}{\sqrt{2}} (f^\dagger_\uparrow c^\dagger_\downarrow - c^\dagger_\uparrow f^\dagger_\downarrow)|0\rangle, \qquad |f^2\rangle = f^\dagger_\uparrow f^\dagger_\downarrow |0\rangle \qquad \text{and} \qquad |c^2\rangle = c^\dagger_\uparrow c^\dagger_\downarrow |0\rangle$$

$$\langle c^2 | \sum_\sigma V c^\dagger_\sigma f_\sigma | f^1 c^1 \rangle = \sqrt{2} V \qquad \langle f^2 | \sum_\sigma V f^\dagger_\sigma c_\sigma | f^1 c^1 \rangle = \sqrt{2} V$$

  - Effective interaction

$$H_K = -2J \left[\frac{1}{4} - \frac{1}{2} \sigma(0) \cdot \mathbf{S}_f \right] \qquad \sigma(0) = \frac{1}{\mathcal{N}} \sum_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}\alpha} \sigma_{\alpha\beta} c_{\mathbf{k}'\beta}$$

  - Effective interaction induced by the virtual charge fluctuations

$$H_K = J \sigma(0) \cdot \mathbf{S}_f \quad \Longrightarrow \quad H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \sigma(0) \cdot \mathbf{S}_f$$

Local moments and the Kondo lattice

$$H(\Lambda) \to UH(\Lambda)U^\dagger = \left[\begin{array}{c|c} \tilde{H}_L & 0 \\ \hline 0 & \tilde{H}_H \end{array}\right]$$

$$H = \left[\begin{array}{c|c} H_L & V^\dagger \\ \hline V & H_H \end{array}\right]$$

- **Hierarchies of energy scales**
  - The Kondo Effect

$$\delta H_{ab} = \langle a|\delta H|b\rangle = \frac{1}{2}\left[T_{ab}(E_a) + T_{ab}(E_b)\right]$$

$$T_{ab}(\omega) = \sum_{|\Lambda\rangle\in\{H\}} \left[\frac{V^\dagger_{a\Lambda}V_{\Lambda b}}{\omega - E_\Lambda}\right] \qquad V = \mathcal{P}_H J\mathbf{S}(0)\cdot\mathbf{S}_d\mathcal{P}_L$$

  - Process I

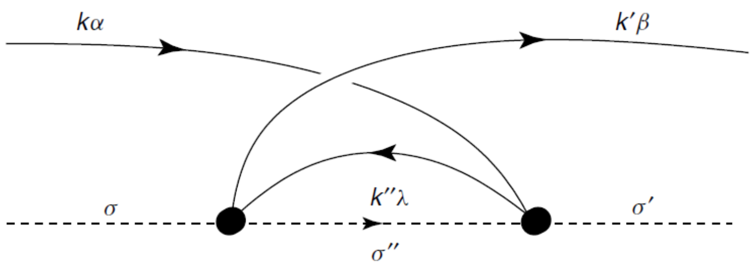
$$\langle k'\beta,\sigma'|\hat{T}^{\mathrm{I}}(E)|k\alpha,\sigma\rangle = \sum_{\epsilon_{k''}\in[\Lambda-\delta\Lambda,\Lambda]} \left[\frac{1}{E-\epsilon_{k''}}\right] J^2(\sigma^a_{\beta\lambda}\sigma^b_{\lambda\alpha})(S^a_{\sigma'\sigma''}S^b_{\sigma''\sigma})$$

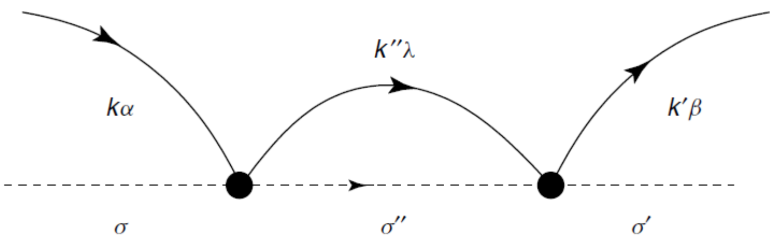
$$\approx J^2\rho\delta\Lambda\left[\frac{1}{E-\Lambda}\right](\sigma^a\sigma^b)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$

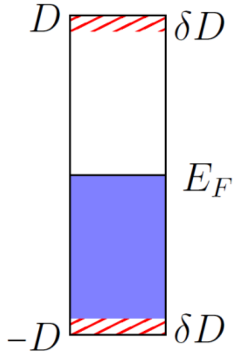
  - Process II

$$\langle k'\beta\sigma'|T^{\mathrm{II}}(E)|k\alpha\sigma\rangle = -\sum_{\epsilon_{k''}\in[-\Lambda,-\Lambda+\delta\Lambda]} \left[\frac{1}{E-(\epsilon_k+\epsilon_{k'}-\epsilon_{k''})}\right] J^2(\sigma^b\sigma^a)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$

$$= -J^2\rho\delta\Lambda\left[\frac{1}{E-\Lambda}\right](\sigma^a\sigma^b)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$







Local moments and the Kondo lattice

- **Hierarchies of energy scales**
  - Combining processes I and II

$$\delta H^{\text{int}}_{k'\beta\sigma';k\alpha\sigma} = \hat{T}^{\text{I}} + \hat{T}^{\text{II}} = -\frac{J^2\rho\delta\Lambda}{\Lambda}\left[\sigma^a,\sigma^b\right]_{\beta\alpha} S^a S^b$$
$$= 2\frac{J^2\rho\delta\Lambda}{\Lambda}\sigma_{\beta\alpha}\cdot\mathbf{S}_{\sigma'\sigma}$$

  - "Anti-screening" of the Kondo coupling constant

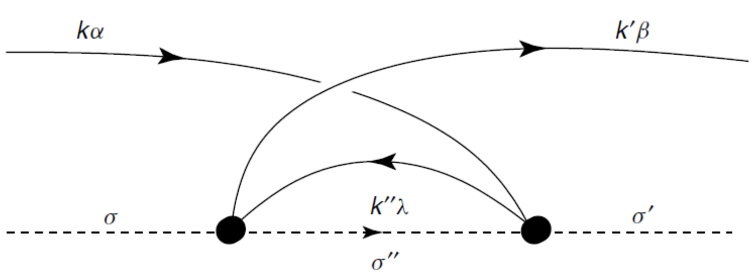
$$J(\Lambda') = J(\Lambda) + 2J^2\rho\frac{\delta\Lambda}{\Lambda}$$

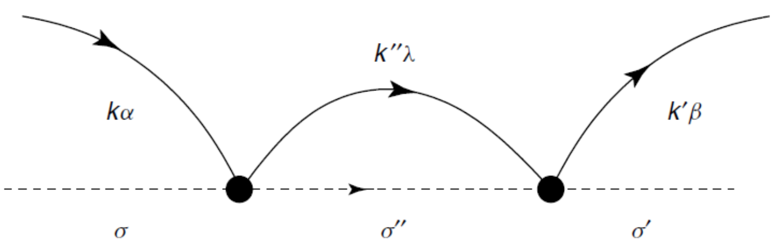
  - Introduce coupling constant $g = \rho J$

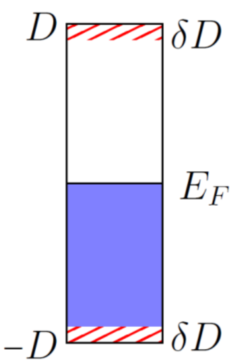
$$\frac{\partial g}{\partial \ln\Lambda} = \beta(g) = -2g^2 + O(g^3)$$

  - Define

$$T_K = D\exp\left[-\frac{1}{2g_o}\right] \quad \Longrightarrow \quad 2g(\Lambda') = \frac{1}{\ln(\Lambda/T_K)}$$







Local moments and the Kondo lattice

- **Hierarchies of energy scales**
  - Kondo temperature is the *only* intrinsic scale

$$F(T) = T_K \Phi\left(\frac{T}{T_K}\right)$$

  - Resistivity

$$\rho_i = \frac{ne^2}{m}\tau(T,H)$$

  - Universal scattering rate

$$\tau(T,H) = \frac{n_i}{\rho}\Phi_2\left(\frac{T}{T_K}, \frac{H}{T_K}\right)$$

  - To leading order in Born approximation; g_0 is bare coupling

$$\tau = 2\pi\rho J^2 S(S+1) = \frac{2\pi S(S+1)}{\rho} g_0^2$$

  - At finite temperatures $g_0 \rightarrow g(\Lambda = 2\pi T)$

$$\tau(T) = \frac{2\pi S(S+1)}{\rho} \frac{1}{4\ln^2(2\pi T/T_K)}$$

  - Formation of spin-singlet bound state at large coupling

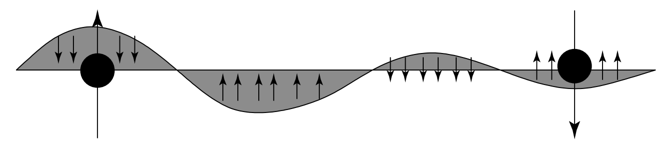
$$J\rho \gg 1 \qquad n_f = 1 \qquad \delta_\uparrow = \delta_\downarrow = \pi/2$$

Local moments and the Kondo lattice

- **Hierarchies of energy scales**
  - Doniach's Kondo lattice concept

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \sum_{j} \mathbf{S}_j \cdot \left(c^{\dagger}_{\mathbf{k}\alpha} \sigma_{\alpha\beta} c_{\mathbf{k}'\beta} \right) e^{i(\mathbf{k}'-\mathbf{k})\cdot \mathbf{R}_j}$$

  - No Kondo physics ➔ Friedel oscillations



$$\langle \sigma(\mathbf{x}) \rangle = -J\chi(\mathbf{x}-\mathbf{x}_0)\langle \mathbf{S}(\mathbf{x}_0)\rangle$$

$$\chi(\mathbf{x}) = 2\sum_{\mathbf{k},\mathbf{k}'} \left(\frac{f(\epsilon_{\mathbf{k}}) - f(\epsilon_{\mathbf{k}'})}{\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}}} \right) e^{i(\mathbf{k}-\mathbf{k}')\cdot \mathbf{x}}$$

$$\langle \sigma(r) \rangle \sim -J\rho \frac{\cos(2k_F r)}{|k_F r|^3}$$

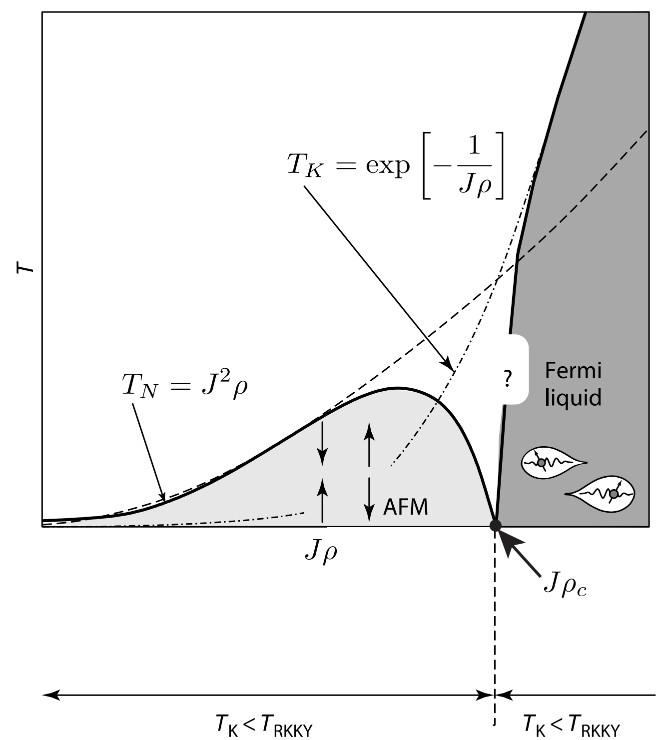
  - Ruderman-Kittel-Kasuya-Yosida (RKKY)

$$H_{\mathrm{RKKY}} = \overbrace{-J^2 \chi(\mathbf{x}-\mathbf{x}')}^{J_{\mathrm{RKKY}}(\mathbf{x}-\mathbf{x}')} \mathbf{S}(\mathbf{x}) \cdot \mathbf{S}(\mathbf{x}')$$

$$J_{\mathrm{RKKY}}(r) \sim -J^2 \rho \frac{\cos(2k_F r)}{k_F r}$$

  - Two possible ordering temperatures

$$T_K = De^{-1/(2J\rho)} \qquad T_{\mathrm{RKKY}} = J^2 \rho$$



Local moments and the Kondo lattice

- **The Large *N* Kondo Lattice**
  - Gauge theories, Large *N* and strong correlation
  - Spin operator representation

$$\mathbf{S}_j = f^\dagger_{j\alpha}\left(\frac{\sigma}{2}\right)_{\alpha\beta} f_{j\beta}$$

$$f_j \to e^{i\phi_j} f_j$$

  - Slave boson approach

$$X_{\sigma 0}(j) = f^\dagger_{j\sigma} b_j, \qquad X_{0\sigma}(j) = b^\dagger_j f_{j\sigma}$$

$$f^\dagger_{j\sigma}|0\rangle \equiv |f^1, j\rangle \qquad b^\dagger_j|0\rangle = |f^0, j\rangle$$

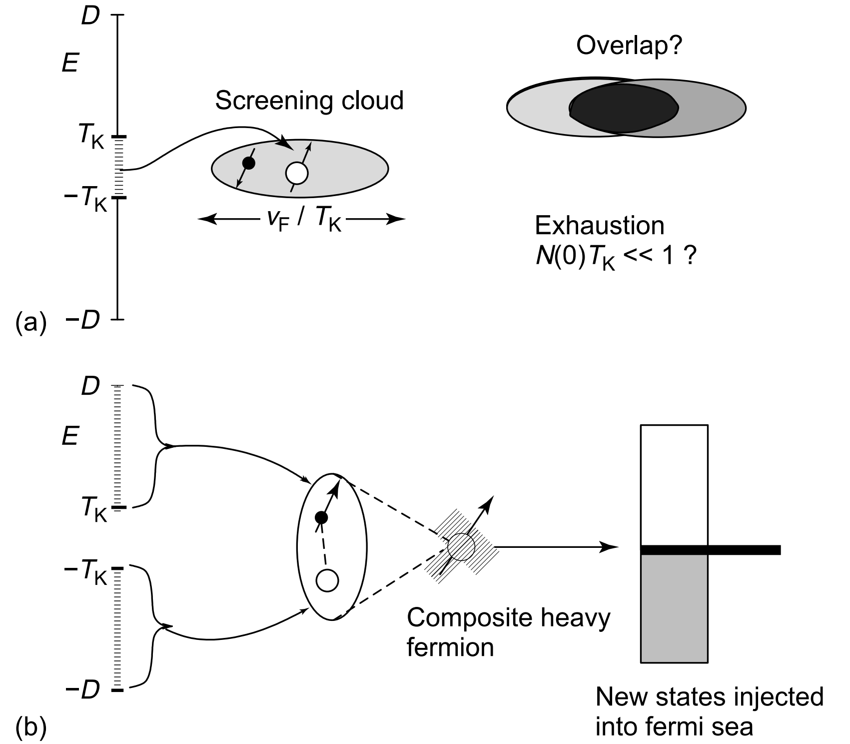
  - Conservation of gauge charge

$$Q_j = \sum_\sigma f^\dagger_{j\sigma} f_{j\sigma} + b^\dagger_j b_j$$

$$f_{j\sigma} \to e^{i\theta_j} f_{j\sigma} \qquad b_j \to e^{i\theta_j} b_j$$

  - Mobility of *f*-states

$$2\frac{\mathcal{V}_{FS}}{(2\pi)^3} = n_e + n_{\text{spins}}$$



Local moments and the Kondo lattice

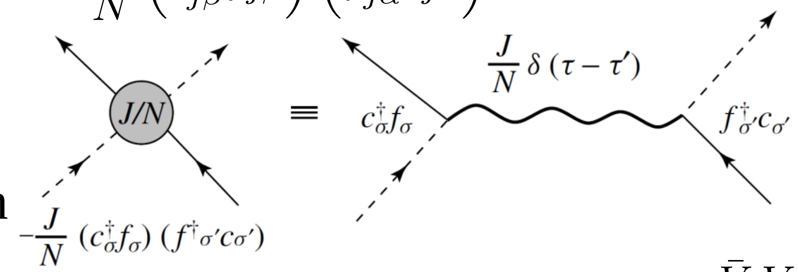
- **The Large *N* Kondo Lattice**
  - Mean field theory of the Kondo lattice

$$Z = \int \mathcal{D}[\phi] e^{-NS[\phi,\dot{\phi}]}$$

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_j H_{\mathrm{I}}(j) \qquad H_{\mathrm{I}}(j) = \frac{J}{N} S_{\alpha\beta}(j) c^\dagger_{j\beta} c_{j\alpha}$$

$$S_{\alpha\beta}(j) = f^\dagger_{j\alpha} f_{j\beta} - \frac{n_f}{N}\delta_{\alpha\beta} \quad \Longrightarrow \quad H_{\mathrm{I}}(j) = -\frac{J}{N}\left(c^\dagger_{j\beta} f_{j\beta}\right)\left(f^\dagger_{j\alpha} c_{j\alpha}\right)$$

$$c^\dagger_{j\alpha} = \frac{1}{\sqrt{\mathcal{N}_s}} \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} e^{-i\mathbf{k}\cdot\mathbf{R}_j}$$



  - Hubbard-Stratonovich transformation

$$-gA^\dagger A \to A^\dagger V + \bar{V} A + \frac{\bar{V}V}{g} \qquad H_{\mathrm{I}}(j) \to H_{\mathrm{I}}[V,j] = \bar{V}_j\left(c^\dagger_{j\alpha} f_{j\alpha}\right) + \left(f^\dagger_{j\alpha} c_{j\alpha}\right) V_j + N\frac{\bar{V}_j V_j}{J}$$

  - Path integral

$$Z = \int \mathcal{D}[V,\lambda] \overbrace{\int \mathcal{D}[c,f] \exp\left[-\int_0^\beta \left(\sum_{k\sigma} c^\dagger_{\mathbf{k}\sigma}\partial_\tau c_{\mathbf{k}\sigma} + \sum_{j\sigma} f^\dagger_{j\sigma}\partial_\tau f_{j\sigma} + H[V,\lambda]\right)\right]}^{=\mathrm{Tr}\left[T\exp\left(-\int_0^\beta H[V,\lambda]d\tau\right)\right]}$$

$$H[V,\lambda] = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_j \left(H_{\mathrm{I}}[V_j,j] + \lambda_j[n_f(j) - Q]\right)$$

Local moments and the Kondo lattice

- **The Large *N* Kondo Lattice**
  - At the saddle point

$$Z = \mathrm{Tr}\left[e^{-\beta H_{\mathrm{MFT}}}\right], \qquad (N \to \infty)$$

$$H_{\mathrm{MFT}} = H[V,\lambda] = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{j,\alpha} \left(\bar{V} c^{\dagger}_{j\beta} f_{j\beta} + V f^{\dagger}_{j\alpha} c_{j\alpha} + \lambda f^{\dagger}_{j\alpha} f_{j\alpha}\right) + Nn\left(\frac{\bar{V}V}{J} - \lambda_o q\right)$$

  - Matrix form of the Hamiltonian

$$H_{\mathrm{MFT}} = \sum_{\mathbf{k}\sigma} \left(c^{\dagger}_{\mathbf{k}\sigma}, f^{\dagger}_{\mathbf{k}\sigma}\right) \begin{bmatrix} \epsilon_{\mathbf{k}} & \bar{V} \\ V & \lambda \end{bmatrix} \begin{pmatrix} c_{\mathbf{k}\sigma} \\ f_{\mathbf{k}\sigma} \end{pmatrix} + Nn\left(\frac{\bar{V}V}{J} - \lambda q\right)$$

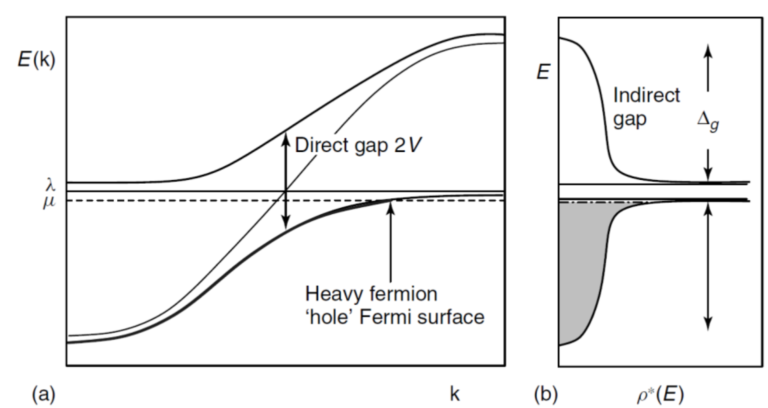
where

$$f^{\dagger}_{\mathbf{k}\sigma} = \frac{1}{\sqrt{\mathcal{N}}} \sum_{j} f^{\dagger}_{j\sigma} e^{i\mathbf{k}\cdot\mathbf{R}_j}$$

  - Diagonalize the Hamiltonian

$$H_{\mathrm{MFT}} = \sum_{\mathbf{k}\sigma} \left(a^{\dagger}_{\mathbf{k}\sigma}, b^{\dagger}_{\mathbf{k}\sigma}\right) \begin{bmatrix} E_{\mathbf{k}+} & 0 \\ 0 & E_{\mathbf{k}-} \end{bmatrix} \begin{pmatrix} a_{\mathbf{k}\sigma} \\ b_{\mathbf{k}\sigma} \end{pmatrix} + N\mathcal{N}_s\left(\frac{|V|^2}{J} - \lambda q\right)$$

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2}\right)^2 + |V|^2\right]^{\frac{1}{2}}$$



Local moments and the Kondo lattice

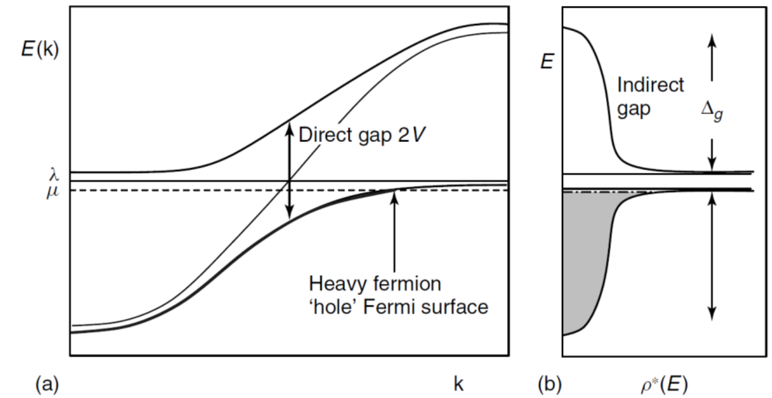
- **The Large *N* Kondo Lattice**
  - "Heavy-fermion" dispersion

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}}$$

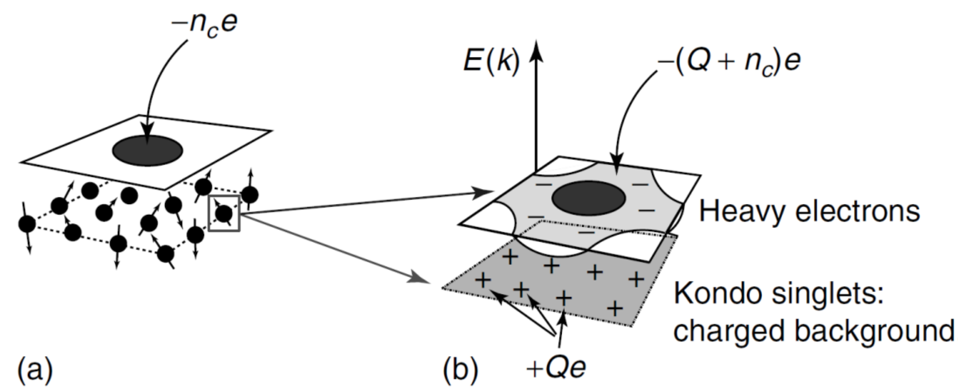
$$\rho^*(E) = \sum_{\mathbf{k},\pm} \delta(E - E_{\mathbf{k}}^{(\pm)})$$

$$\rho^*(E) = \rho \frac{d\epsilon}{dE} = \rho(\epsilon) \left(1 + \frac{|V|^2}{(E-\lambda)^2} \right) \sim \begin{cases} \rho \left(1 + \frac{|V|^2}{(E-\lambda)^2} \right) & \text{outside hybridization gap,} \\ 0 & \text{inside hybridization gap,} \end{cases}$$

$$\overbrace{n_e}^{e^- \text{density}} = \overbrace{N \frac{V_{FS}}{(2\pi)^3}}^{\text{q. particle density}} - \underbrace{\frac{Q}{a^D}}_{+\text{ve background}}$$



  - How to find *V* and λ?
  - Compute free energy

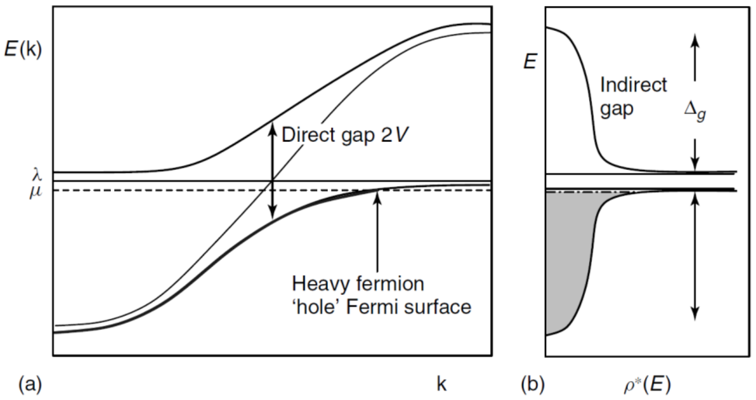


Local moments and the Kondo lattice

- **The Large *N* Kondo Lattice**
  - "Heavy-fermion" dispersion

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2}\right)^2 + |V|^2\right]^{\frac{1}{2}}$$

$$\rho^*(E) = \sum_{\mathbf{k},\pm} \delta(E - E_{\mathbf{k}}^{(\pm)})$$



$$\rho^*(E) = \rho\frac{d\epsilon}{dE} = \rho(\epsilon)\left(1 + \frac{|V|^2}{(E-\lambda)^2}\right) \sim \begin{cases} \rho\left(1 + \frac{|V|^2}{(E-\lambda)^2}\right) & \text{outside hybridization gap,} \\ 0 & \text{inside hybridization gap,} \end{cases}$$

  - Compute free energy

$$\frac{F}{N} = -T\sum_{\mathbf{k},\pm} \ln\left[1 + e^{-\beta E_{\mathbf{k}\pm}}\right] + \mathcal{N}_s\left(\frac{|V|^2}{J} - \lambda q\right)$$

$$\frac{E_0}{N\mathcal{N}_s} = \int_{-\infty}^{0} \rho^*(E)E + \left(\frac{|V|^2}{J} - \lambda q\right)$$

$$\frac{E_o}{N\mathcal{N}_s} = \int_{-D}^{0} d\epsilon\rho E dE + \int_{-D}^{0} dE\rho|V|^2\frac{E}{(E-\lambda)^2} + \left(\frac{|V|^2}{J} - \lambda q\right)$$

$$= \overbrace{-\frac{D^2\rho}{2}}^{E_c/(N\mathcal{N}_s)} + \overbrace{\frac{\Delta}{\pi}\ln\left(\frac{\lambda e}{T_K}\right) - \lambda q}^{E_K/(N\mathcal{N}_s)}$$

$$\frac{\partial E_o}{\partial \lambda} = 0 \implies \lambda = \frac{\Delta}{\pi q}$$

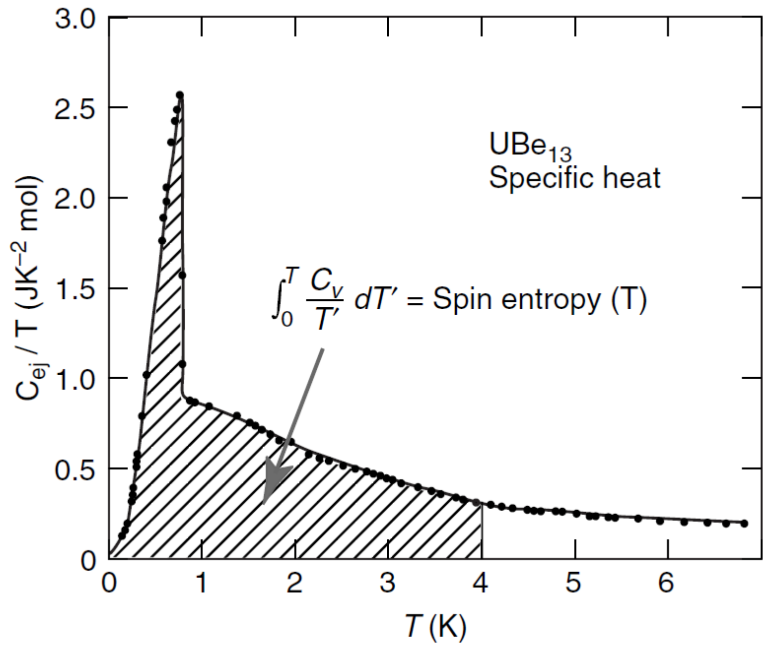
$$\frac{E_K}{N\mathcal{N}_s} = \frac{\Delta}{\pi}\ln\left(\frac{\Delta e}{\pi q T_K}\right)$$

$$\Delta = \frac{\pi q}{e^2}T_K$$

$$\rho^*(0) = \rho + \frac{q}{T_K}$$

$$\frac{m^*}{m} = 1 + \frac{q}{\rho T_K} \sim \frac{qD}{T_K}$$

Heavy-fermion superconductivity



- **Phenomenology**
  - Spin entropy

$$\frac{(C_\mathrm{v}^\mathrm{s} - C_\mathrm{v}^\mathrm{n})}{C_\mathrm{v}} \sim 1\text{-}2$$

$$\int_0^{T_c} dT \frac{(C_\mathrm{v}^\mathrm{s} - C_\mathrm{v}^\mathrm{n})}{T} = 0$$

  - London penetration depth agrees with enhanced mass

$$\frac{1}{\mu_o \lambda_L^2} = \frac{ne^2}{m^*} = \int_{\omega \in \mathrm{D.P}} \frac{d\omega}{\pi} \sigma(\omega) \ll \frac{ne^2}{m}$$

  - Coherence length reduces
  - "BCS-like" theory

$$H = \sum_{\mathbf{k},\sigma} \epsilon_\mathbf{k} f^\dagger_{\mathbf{k}\alpha} f_{\mathbf{k}\alpha} + \sum_\mathbf{k} \left[f^\dagger_{\mathbf{k}\alpha} \Delta_{\alpha\beta}(\mathbf{k}) f^\dagger_{-\mathbf{k}\beta} + f_{-\mathbf{k}\beta} \bar{\Delta}_{\beta\alpha}(\mathbf{k}) f_{\mathbf{k}\alpha} \right]$$

$$E_\mathbf{k} = \sqrt{\epsilon_\mathbf{k}^2 + |\Delta_\mathbf{k}|^2} \qquad \Delta_{\alpha\beta}(\mathbf{k}) = \begin{cases} \Delta(\mathbf{k})(i\sigma_2)_{\alpha\beta} & \text{(singlet)}, \\ \mathbf{d}(\mathbf{k}) \cdot (i\sigma_2 \boldsymbol{\sigma})_{\alpha\beta} & \text{(triplet)}. \end{cases}$$

  - Example: UPt_3 $\quad \Delta_\mathbf{k} \propto k_z(\hat{k}_x \pm i k_y), \qquad |\Delta_\mathbf{k}|^2 \propto k_z^2(k_x^2 + k_y^2)$

Heavy-fermion superconductivity

- **Phenomenology**
  - Density of states near line nodes

$$N^*(E) = 2\sum_{\mathbf{k}} \delta(E - E_{\mathbf{k}}) \propto E$$

$$E_{\mathbf{k}} \sim \sqrt{(v_F k_1)^2 + (\alpha k_2)^2} \qquad \alpha = \frac{d\Delta}{dk_2}$$

  - Specific heat

$$\frac{C_V}{T} \propto \overbrace{\overline{N(E)}}^{\propto T} \sim T$$

  - NMR

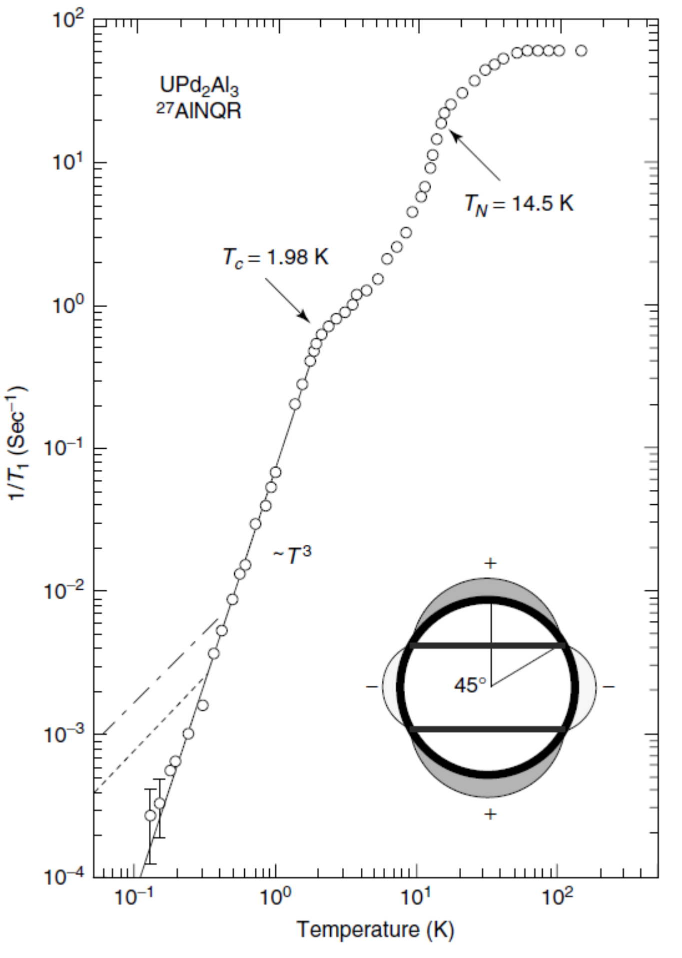
$$\frac{1}{T_1} \propto T \overbrace{\overline{N(E)^2}}^{\propto T^2} \sim T^3$$

  - "Volovik effect"

$$E_{\mathbf{k}} \to E_{\mathbf{k}} + \mathbf{p}\cdot\mathbf{v}_s = E_{\mathbf{k}} + \mathbf{v}_F \cdot \frac{\hbar}{2}\nabla\phi$$

$$\Delta E \sim \hbar\frac{v_F}{2R} \qquad \Phi_0 = \frac{h}{2e} \qquad \pi H R^2 \sim \Phi_0 \qquad \pi H_{c2}\xi^2 \sim \Phi_0 \qquad \xi \sim \frac{v_F}{\Delta}$$

$$E_H \sim \Delta\sqrt{\frac{H}{H_{c2}}} \qquad \frac{1}{R} \sim \frac{1}{\xi}\sqrt{\frac{H}{H_{c2}}}$$



Heavy-fermion superconductivity

- **Phenomenology**
  - "Volovik" effect

$$E_{\mathbf{k}} \to E_{\mathbf{k}} + \mathbf{p}\cdot\mathbf{v}_s = E_{\mathbf{k}} + \mathbf{v}_F \cdot \frac{\hbar}{2}\nabla\phi$$

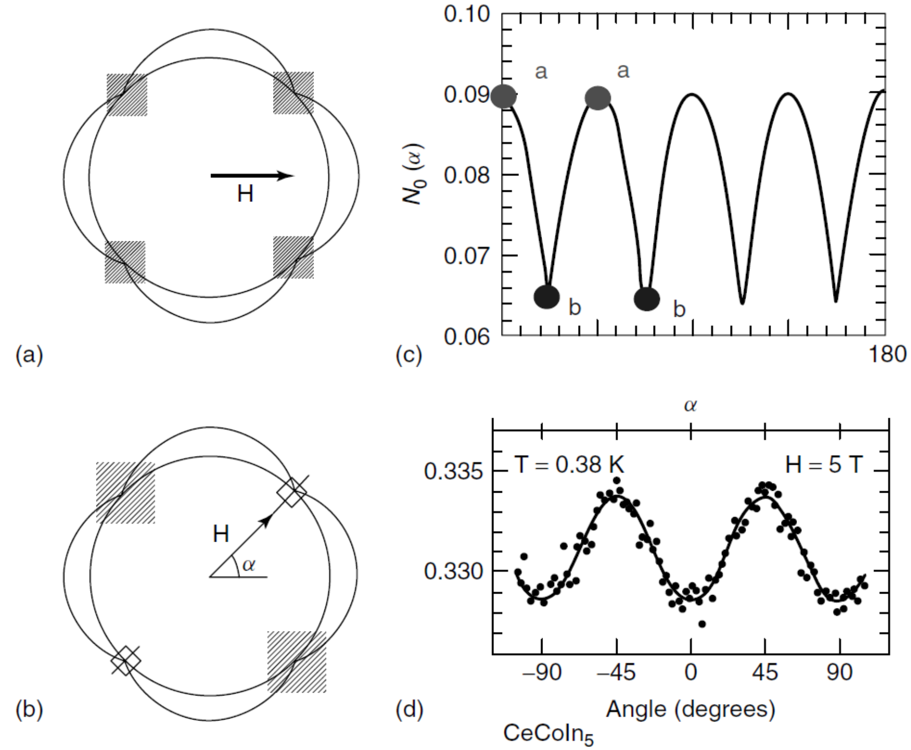
$$\Delta E \sim \hbar\frac{v_F}{2R} \qquad \Phi_0 = \frac{h}{2e} \qquad \pi H R^2 \sim \Phi_0 \qquad \pi H_{c2}\xi^2 \sim \Phi_0 \qquad \xi \sim \frac{v_F}{\Delta}$$

$$E_H \sim \Delta\sqrt{\frac{H}{H_{c2}}} \qquad \frac{1}{R} \sim \frac{1}{\xi}\sqrt{\frac{H}{H_{c2}}}$$

  - Density of quasiparticle states

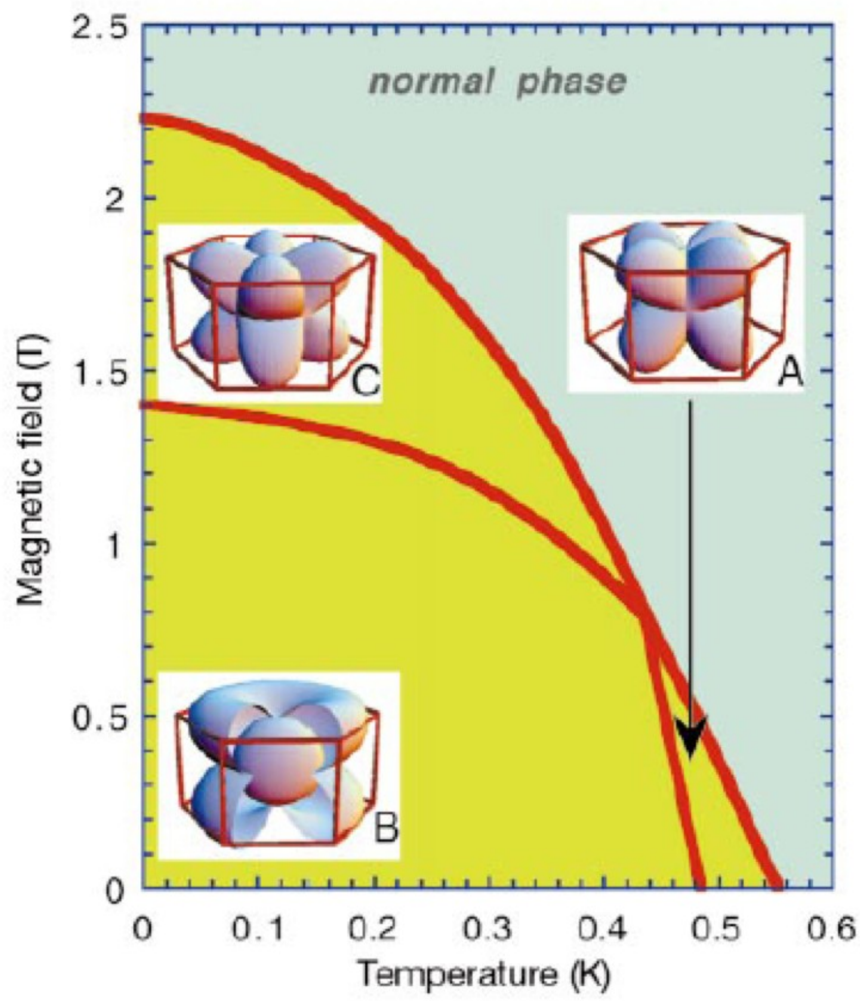
$$N^*(H) \sim N(0)\sqrt{\frac{H}{H_{c2}}}$$

  - DOS depends maximum when magnetic field and node perpendicular



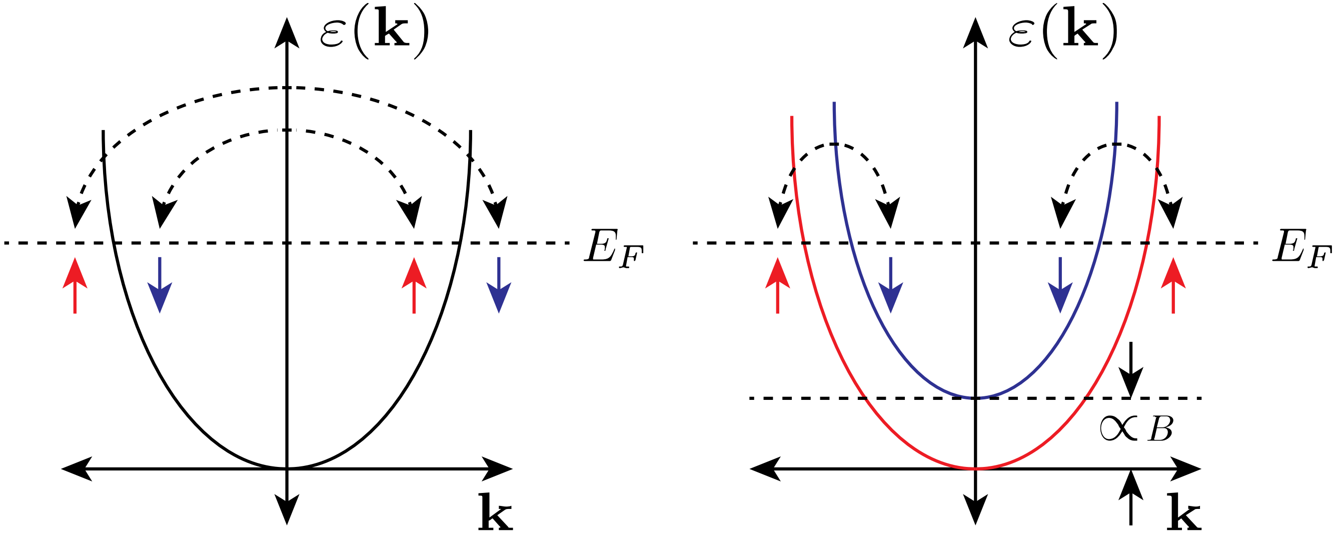
Superconducting "phases" of UPt_3

- **Symmetry breaking in superconductor**
  - Three superconducting phases
  - $T_N = 5$ K
  - $T_c = 0.54$ K
  - Antiferromagnetic order is instrumental for the symmetry breaking between the different superconducting phases



Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) state

- **FFLO properties**
  - Superconductor in a large Zeeman field (B)
  - Cooper pairs have **finite** momentum



  - **Spatially** oscillating order parameter
  - Proposed candidate: $CeCoIn_5$
  - Unfortunately even the slightest disorder destroys the FFLO state