FQHE Dec. 3

> Recap Chern-Simons Theory

$$\mathcal{L} = -\frac{m}{4\pi}\varepsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + j^\mu a^\mu$$

a: "internal" gauge field

j: matter (quasiparticles)

Nick showed that this has:

- topological order (g.s. degeneracy, fractionalization)
- quantized "Hall conductivity"

> Outline

- connect to "external" gauge field ≡ EM field
- aka connect to physical QHE
- hierarchical construction & K-matrix theory
- edge states

> Connection to Physical QHE

- We want an effective theory, which will work at low energies (complete theory too complicated)
- we guess the theory, then see if it is right by comparing to other calculations

$$\mathcal{L} = \underset{\substack{\uparrow \\ \text{Maxwell term}}}{F_{\mu\nu}F^{\mu\nu}} + \frac{e}{2\pi}\underset{\substack{\uparrow \\ \text{charged matter}}}{J^\mu} A^\mu + S[J]$$

- introduce internal gauge field (trick) $\frac{J^\mu}{e} = \frac{1}{2\pi}\varepsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$ ← internal gauge field
- we can do this because J are conserved
- "guess" S[J] to be Chern-Simons term

$$\mathcal{L} = FF + \frac{e}{2\pi}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda - \frac{m}{4\pi}\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$

$$\frac{\partial\mathcal{L}}{\partial a_\mu} = \frac{e}{2\pi}\varepsilon^{\mu\nu\lambda}\partial_\nu A_\lambda - \frac{m}{4\pi}\varepsilon^{\mu\nu\lambda}\partial_\nu a_\lambda = 0$$

$$\therefore \frac{eB}{2\pi} = \frac{m}{e}J$$

$$\therefore J = \left(\frac{1}{m}\right)\frac{e^2}{h}B$$

The guess works!!

~~Quasiparticles~~

~~- add lja term~~

~~- j is "quasiparticle current" NOT electron current~~

~~charge:~~

Properties of $a\partial a$ term: "Hopf term"

$$2\left(\frac{2\pi}{e}\right)^2 \pi m\, J_1^\mu \left(\frac{1}{\partial_\lambda \varepsilon^{\mu\nu\lambda}}\right) J_2^\nu = \int d^3x\, J_1^\mu f_\mu = \oint f_\mu = \int_S \partial f = \int_S \delta(0)$$

let $J_1 = \delta(x_1 - \vec{x})$

$J_2 = \delta(0)$

$\partial f_\mu = J_2$

$\oint 1_0$

$= 2\pi m : f$

path of J_1 encircles J_2

- so this is just saying that J particles have fermionic statistics (which makes sense since they are fermions)
- An action in terms of free electrons has reproduced fractional Hall effect!
- But not: fractional charge & statistics, more complicated fractions

> Quasiparticles

add $l a^\mu j^\mu$ to action

j is quasiparticle current, carries charge of a field

To get properties, integrate out a

$$S = \pi \frac{l^2}{m} j^\mu \left(\frac{1}{\partial_\lambda \varepsilon^{\mu\lambda\nu}}\right) j^\nu + \frac{el}{m} j^\mu A^\mu + \frac{e^2}{m} A \partial A + \text{Maxwell term}$$

1) $\Theta = \frac{2\pi l^2}{m}$

2) $q = \frac{el}{m}$

3) CS in external fields

4) Maxwell term irrelevant → coupling has mass dimension -1, but ignore - mass dimensions at low energies

$a\partial a$ most relevant term allowed by symmetry & gauge invariance

More complicated fractions (Hierarchial construction)

let $j = \epsilon\partial\tilde{a}$

$$\mathcal{L} = \frac{e}{2\pi} A\partial a - \frac{m_1}{4\pi} a\partial a + \frac{1}{2\pi} a\partial\tilde{a} - \frac{m_2}{4\pi}\tilde{a}\partial\tilde{a}$$

- j is a boson, so m_2 is even
- can minimize w.r.t. a & $\tilde{a}$ to get Hall conductivity
- can integrate out a & $\tilde{a}$ to get charge & statistics

Has been done for us: K-matrix theory

$$\mathcal{L} = -\frac{1}{4\pi} K_{IJ}\, a_I \partial a_J + \frac{e}{2\pi} q_I A_\mu \partial a_{I\nu}$$

where $K_{IJ} = \begin{bmatrix} m_1 & -1 \\ -1 & m_2 \end{bmatrix}$ $q_I = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Excitations can carry charge of a & $\tilde{a}$: $j' l_I a_I$

Can actually include more terms

$$\sigma = q^T K^{-1} q \quad Q = e q^T K^{-1} l \quad \Theta = \pi l^T K^{-1} l$$

- reproduces experimentally observed fractions
- can also deal with multilayer states this way

Edge States

- CS term not gauge invariant, claim that they are physical
- Pick them to give right results
- Try $a_0 = 0$, $a_i = \partial_i \phi$

$$S = -\frac{m}{4\pi}\int \partial_t(\partial_x\phi)(\partial_y\phi) \qquad \text{But } \partial_y\phi = \phi(\text{vacuum}) - \phi(\text{QHF}) = \phi(\text{QHF})$$

$$= -\frac{m}{4\pi}\int \partial_t\phi\,\partial_x\phi$$

$H = 0$, velocity zero, wrong

- Try $a_0 = -v a_x$

$$S = -\frac{m}{4\pi}\int \partial_t\phi\,\partial_x\phi + v(\partial_x\phi)^2$$

$H = v(\partial_x\phi)^2 \rightarrow$ canonically quantize: $H = 2\pi m v \rho_k^\dagger \rho_k$

$$[\rho_k, \rho_{k'}] = \frac{k\,\delta_{kk'}}{2\pi M} \quad \text{"Kac-Moody algebra"}$$

only works when $vM < 0$

More generally:

$$S = \frac{1}{4\pi}\int K_{IJ}\,\partial_t\phi_I\,\partial_x\phi_J - V_{IJ}\,\partial_x\phi_I\,\partial_x\phi_J$$

- agrees with hydrodynamical calculations