Topological Field Theory of Time-Reversal invariant insulators III

Review of the last 2 weeks

- It is very very hard to do anything with the microscopic Hamiltonian for topological systems
- Instead we use effective theory
- Quantum Hall states $\longrightarrow \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$
- This paper: topological insulators $\longrightarrow \theta E \cdot B$

How do we know this is correct?

- Shu-ping: This effective theory produces half-integer quantum Hall effect, consistent with experiment
- Tejas: Propose Faraday rotation experiment
- Is this really enough evidence?
- Tejas: this effective action is the dimensional reduction of a second Chern-Simons term
- Claim: Dimensional reduction on a topological phase leads to a topological phase in one less dimension
- Section II (not discussed): dimensional reduction on first Chern-Simons term gives Goldstone-Wilczek formula for charge pumping

This talk

- Sec IV/V: This dimensional reduction reproduces the symmetry classifications from the literature
- Sec V: Further dimensional reduction to 2+1D gives Quantum Spin Hall effect
- Sec VI: Why this scheme can't be used to generate other TIs

Symmetry Classification

- Dimensional reduction $h(k_x, k_y, k_z, k_w) \rightarrow h(k_x, k_y, k_z, \theta)$
- Topological phases classified by second chern number: integral of berry curvature
- Impose time-reversal symmetry: only $\theta = 0$ or π physical
- Multiple hamiltonians lead to the same physical system
- Is Chern number still an invariant?

Symmetry Classification

$$C_2[h_1] - C_2[h_2] = C_2[g_1] + C_2[g_2] \tag{1}$$

$$g_1(k,\theta) = \begin{cases} h_1(k,\theta) & 0 < \theta < \pi \\ h_2(k, 2\pi - \theta) & \pi < \theta < 2\pi \end{cases} \tag{2}$$

$$g_2(k,\theta) = \begin{cases} h_1(k,\theta) & \pi < \theta < 2\pi \\ h_2(k, 2\pi - \theta) & 0 < \theta < \pi \end{cases} \tag{3}$$

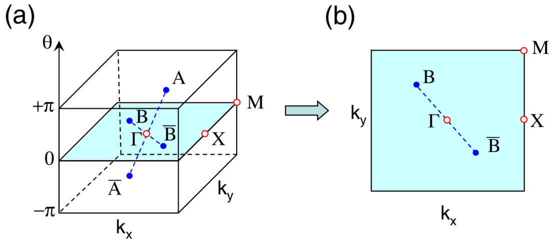
g_1 and g_2 have the same eigenstates, leads to

$$C_2[g_1] = C_2[g_2] \tag{4}$$

$$C_2[h_1] - C_2[h_2] = 2N \tag{5}$$

Symmetry Classification

- From section III: # of surface states in 4+1D=Chern number
- Add time-reversal symmetry and if Chern number is odd, odd # of nodes at time-reversal-symmetric points
- Therefore # of surface states for 3+1D has same parity as Chern number
- Non-trivial 3+1D TI has odd # of Dirac cones!



(3+1)d surface of (4+1)d system

(2+1)-d surface of (3+1)d system

Dimensional reduction to 2+1 D

The calculation is a repeat of last week, except 2 momenta are replaced by parameters

$$
\begin{aligned}
H[A] = & \sum_{k_z,k_w,\mathbf{x}} \sum_{s=1,2} \left[\psi^\dagger_{\mathbf{x};k_z,k_w} \left(\frac{c\Gamma^0 - i\Gamma^s}{2} \right) e^{iA_{\mathbf{x},\mathbf{x}+\hat{s}}} \psi_{\mathbf{x}+\hat{s};k_z,k_w} + \text{H.c.} \right] \\
& + \sum_{k_z,k_w,\mathbf{x}} \sum_{s=1,2} \psi^\dagger_{\mathbf{x};k_z,k_w} \cdot \{ \sin(k_z + A_{\mathbf{x}3}) \Gamma^3 \\
& + \sin(k_w + A_{\mathbf{x}4}) \Gamma^4 + [m + c\cos(k_z + A_{\mathbf{x}3}) \\
& + c\cos(k_w + A_{\mathbf{x}4})] \Gamma^0 \} \psi_{\mathbf{x};k_z,k_w},
\end{aligned}
$$

$$
\begin{aligned}
H_{2\text{D}}[A,\theta,\varphi] = & \sum_{\mathbf{x},s} \left[\psi^\dagger_{\mathbf{x}} \left(\frac{c\Gamma^0 - i\Gamma^s}{2} \right) e^{iA_{\vec{x},\vec{x}+\hat{s}}} \psi_{\mathbf{x}+\hat{s}} + \text{H.c.} \right] \\
& + \sum_{\mathbf{x},s} \psi^\dagger_{\mathbf{x}} [\sin\theta_{\mathbf{x}} \Gamma^3 + \sin\varphi_{\mathbf{x}} \Gamma^4 + (m + c\cos\theta_{\mathbf{x}} \\
& + c\cos\varphi_{\mathbf{x}}) \Gamma^0] \psi_{\mathbf{x}}. \qquad (108)
\end{aligned}
$$

$$
S_{2\text{D}} = \frac{G_2(\theta_0,\varphi_0)}{2\pi} \int d^2x dt \epsilon^{\mu\nu\rho} A_\mu \partial_\nu \delta\theta \partial_\rho \delta\varphi,
$$

Dimensional reduction to 2+1 D

By similar arguments to 3+1 case, there is a $\mathbb{Z}_2$ classification for 2+1 D (suggests QSH)

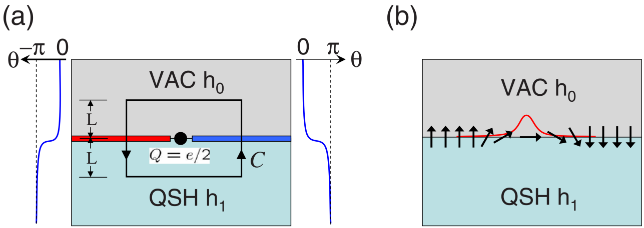
$$S_{2D} = \frac{1}{2\pi} \int d^2x dt \epsilon^{\mu\nu\tau} A_\mu \partial_\nu \Omega_\tau \tag{6}$$

$$j^\mu = \frac{1}{2\pi} \epsilon^{\mu\nu\tau} \partial_\nu \Omega_\tau \tag{7}$$

This action is in terms of the EM gauge field and an integral over the Berry curvature in the θ, ϕ space

Want to argue that this is the effective action for a quantum spin Hall system

Half-charge at domain wall



More QSH

- Break time-reversal symmetry on the edge in opposite ways
- This allows an edge of the system without gapless modes
- Integrate total charge around a loop
- Equivalent to polarization, which is quantized to $1/2$
- $1/2$ charge quantization known from literature
- As in $3+1$D case, odd number of dirac cones $\rightarrow$ gapless edge

- Contains more general prescription for dimensional reduction in any dimensions
- Faster derivations
- We have already discussed all possible realistic TIs

Why are there no more realistic TIs

- Example: try finding 1+1D theory from the QSH
- Need a topological invariant
- Want to find $C_2[h] - C_2[h']$
- This isn't defined because they can also have different C_1