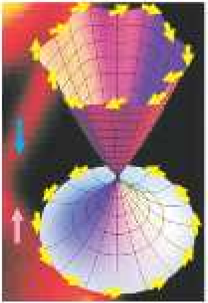
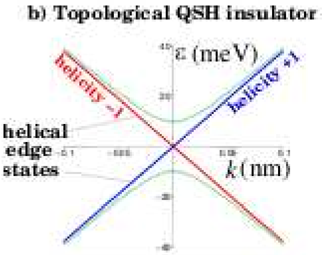
Spin-helical transport in normal and superconducting topological insulators

Introduction

- Spin-momentum locking=helical edge states
- Helical edge states lead to novel transport properties
- But its not as easy to detect as for Quantum Spin Hall
- Requires Fermi energy in the gap



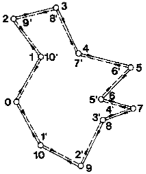


Outline/Novel Transport Properties

- Weak antilocalization
- Fractional Quantum Hall
- Interface with Superconductors

- Hamiltonian: $\mathcal{A}\sigma \cdot k$ term (phenomenological)
- *very strong* spin-orbit coupling leads to $E(k) \approx \mathcal{A}k$
- Impurity protection=diffusive regime=constant spin
- Q: How does large $\mathcal{A}$ protect against impurities and enforce time-reversal symmetry?

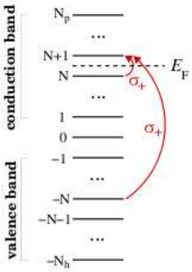
- In diffusion regime, a lot like classical (Drude) insulator
- Spin-orbit coupling leads to destructive interference when backscattered
- Increases conductivity
- Magnetic Field can change the conductivity in well-defined ways
- Due to small penetration of edge state into the bulk, field perpendicular to the surface has a much larger effect than field parallel to the surface

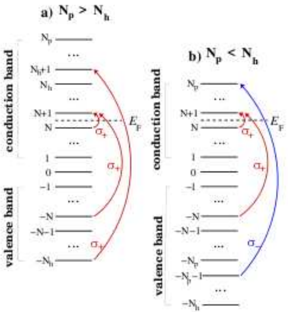


- Take only spin-orbit term of Hamiltonian, add magnetic field
- Q: Why doesn't the strong magnetic field destroy the topological order?
- Can use gauge freedom to set diagonal terms to 0, off-diagonal to $-i\hbar k_x - eBx = a$
- Diagonalize
- Kubo formula: $\langle j_\alpha(t)\rangle = \int dti\theta(t-t')\langle[j_\alpha(t), j_\beta(t')]\rangle A(t')$
- Fourier transform!
- Note that j's are in the interaction picture, and $E = A/\omega$

$$j_\alpha = ev\sigma_\alpha$$
$$\langle \bar{n}'|\sigma_\pm|\bar{n}\rangle = \pm i\frac{1}{2}\delta_{|n'|,|n|\pm 1}$$

- $\sigma_{xy}(\omega) = \sum_n \sum_{n'} (f_n - f_{n'}) \frac{|\langle \bar{n'}|\sigma_+|\bar{n}\rangle|^2 - |\langle \bar{n'}|\sigma_-|\bar{n}\rangle|^2}{(\epsilon_n + \epsilon_{n'})^2/\hbar^2 + (\tau - i\omega)^2}$
- Terms like $-N-1 \to N+2$ and $-N-2 \to N+1$ cancel
- So for equal number of particles and holes, only $N \to N+1$ and $-N \to N+1$ contribute
- Leads to $\sigma_{xy} = \frac{e^2}{\hbar}\left[N + \frac{1}{2}\right]$
- For different number of particles and holes, can have another contribution which breaks up half-integer effect


conduction band
valence band
N_p
N+1
N
E_F
1
0
−1
−N
−N−1
$-N_h$
σ_+
σ_+


a) $N_p > N_h$
b) $N_p < N_h$
conduction band
valence band
N_p
N_h+1
N_h
N+1
N
E_F
1
0
−1
−N
−N−1
$-N_p$
$-N_p-1$
$-N_h$
σ_+
σ_-

- Top and bottom layer can have different N, makes it difficult to observe this effect
- Can also compute AC conductivities

- Novel behaviour expected at S/TI/S junction
- Spin-momentum locking induces antisymmetric p-wave pairing on the superconductor
- Could form Majoranas at interface, but for thin TI these might be able to pair
- Direct calculation confirms existence of p states
- Andreev Bound States (ABS) are states in the junction formed by Andreev reflection
- When the states are perpendicular to the boundary, there is no backscattering
- This leads to different states (and different currents through the junction) than in the non-topological case