Recap: U(1) slave-boson formulation of t-J model and mean field theory

• Mean field phase diagram

\[ H = \sum_{ij} J \left( S_i \cdot S_j - \frac{1}{4} n_i n_j \right) - \sum_{ij} t_{ij} \left( c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.} \right) \]

\[ \chi_{ij} = \sum_\sigma \langle f_{i\sigma}^\dagger f_{j\sigma} \rangle \quad \Delta_{ij} = \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle \quad b = \langle b_i \rangle \]

\[ L_1 = \sum_{i,\sigma} f_{i\sigma}^* \left( \frac{\partial}{\partial \tau} - \mu_F + ia_0(r_i) \right) f_{i\sigma} + \sum_{i} b_i^* \left( \frac{\partial}{\partial \tau} - \mu_B + ia_0(r_i) \right) b_i \]

\[ -\tilde{J} \chi \sum_{ij,\sigma} \left( e^{ia_{ij}} f_{i\sigma}^* f_{j\sigma} + \text{h.c.} \right) - \eta \sum_{ij} \left( e^{ia_{ij}} b_i^* b_j + \text{h.c.} \right) \]

<table>
<thead>
<tr>
<th>Label</th>
<th>State</th>
<th>( \chi )</th>
<th>( \Delta )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Fermi liquid</td>
<td>( \neq 0 )</td>
<td>= 0</td>
<td>( \neq 0 )</td>
</tr>
<tr>
<td>II</td>
<td>Spin gap</td>
<td>( \neq 0 )</td>
<td>( \neq 0 )</td>
<td>= 0</td>
</tr>
<tr>
<td>III</td>
<td>( d )-wave superconducting</td>
<td>( \neq 0 )</td>
<td>( \neq 0 )</td>
<td>( \neq 0 )</td>
</tr>
<tr>
<td>IV</td>
<td>uRVB</td>
<td>( \neq 0 )</td>
<td>= 0</td>
<td>= 0</td>
</tr>
</tbody>
</table>
SU(2) slave-boson formulation of t-J model and mean field theory

- Mean field phase diagram

\[
\psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow} \end{pmatrix}, \quad h_i = \begin{pmatrix} b_{1i} \\ b_{2i} \end{pmatrix}
\]

\[
c_{i\uparrow} = \frac{1}{\sqrt{2}} h_i^\dagger \psi_i = \frac{1}{\sqrt{2}} \left( b_{1i} f_{i\uparrow} + b_{2i} f_{i\downarrow} \right)
\]

\[
c_{i\downarrow} = \frac{1}{\sqrt{2}} h_i^\dagger \bar{\psi}_i = \frac{1}{\sqrt{2}} \left( b_{1i} f_{i\downarrow} - b_{2i} f_{i\uparrow} \right)
\]

\[
\bar{\psi} = i \tau^2 \psi^*
\]

\[
H_{\text{mean}} = \sum_{\langle ij \rangle} \frac{3}{8} J \left[ \frac{1}{2} \text{Tr}(U_{ij}^\dagger U_{ij}) + (\psi_i^\dagger U_{ij}\psi_j + \text{h.c.}) \right]
\]

\[
-\frac{1}{2} \sum_{\langle ij \rangle} t(h_i^\dagger U_{ij} h_j + \text{h.c.}) - \mu \sum_i h_i^\dagger h_i + \sum_i a_0^\dagger (\psi_i^\dagger \tau^l \psi_i + h_i^\dagger \tau^l h_i)
\]
I. Introduction

A. Aim and scope

• Discovery
  • Discovered in 2008 by Kamihara et al. in LaFeAsO with $T_c = 26$ K
  • Similarities with the cuprates
    • High-$T_c$
    • quasi-2D
    • *Almost* universal phase diagram
I. Introduction

B. Fe-based superconductors

• Comparison with cuprates
  • Gap structure unclear after following experiments:
    • Penetration depth
    • ARPES
    • NMR
    • Phase sensitive Josephson tunneling

• Important questions
  • What is the gap structure?
  • What is pairing mechanism?
  • What is the role of disorder in experiments?
I. Introduction

B. Fe-based superconductors

- Comparison with cuprates
  - Fe has weaker interactions
  - $2p$-ligands (e.g. As) lie out of Fe plane
  - Multiple bands near the Fermi energy
  - Undoped or parent compounds are poor metals with SDW order
  - Coexistence of superconductivity and magnetism
  - No robust pseudogap region
  - Doping involves in-plane and out-of-plane substitution, e.g. Sr$_{1-x}$K$_x$Fe$_2$As$_2$, Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$, etc.
I. Introduction

B. Fe-based superconductors

• **Comparison with MgB$_2$**
  - First example of multigap superconductivity
  - $T_c = 40$ K $\Rightarrow$ higher than first cuprate!
  - Type-II multiband BCS superconductor

• **Conceptual importance**
  - *Not* an improvement from practical standpoint
    - $T_c$ lower than cuprates (maximum 55 K)
    - Expensive to fabricate and difficult to work with
  - Pnictides demonstrated cuprate properties are *not* unique to high-$T_c$}
  - Differences in cuprates and pnictides highlight critical ingredients of high-$T_c$
I. Introduction

B. Fe-based superconductors

• Families of pnictides/chalcogenides
  • 1111 family  $\Rightarrow$ LaFeAsO  $\Rightarrow$ LaFeAsO$_{1-x}$F$_x$
  • 122 family  $\Rightarrow$ SrFe$_2$As$_2$
  • 111 family  $\Rightarrow$ LiFeAs
  • 11 family  $\Rightarrow$ FeTe
I. Introduction

B. Fe-based superconductors

• Gap symmetry and structure
  • Experiments ruled out triplet pairing
  • $s$-wave and $d$-wave have different symmetry
  • $s_{++}$ and fully gapped $s_{\pm}$ make sense in multiband picture
  • Gap symmetry $\Rightarrow$ phase change in $k$-space
  • Singlet vs. triplet $\Rightarrow$ phase change in spin-space (no spin-orbit coupling)
  • Gap “structure” $\Rightarrow$ everything else! e.g. phase change across Fermi sheets
II. Electronic structure  

B. Minimal band models  

- Multiband nature  
  - Unit cells $\Rightarrow$ “folded” (blue) and “unfolded” (green)  
  - Treat height of Arsenic as a perturbation  
  - Simplest model in “unfolded” zone  

\[
H = \sum_{\mathbf{k},\sigma,i=\alpha_1,\alpha_2,\beta_1,\beta_2} \varepsilon_{\mathbf{k}}^i \hat{c}_{i\mathbf{k}\sigma} \hat{c}^\dagger_{i\mathbf{k}\sigma} \quad \varepsilon_{\mathbf{k}}^{\alpha_1,2} = -\frac{\mathbf{k}^2}{2m_{1,2}} + \mu \\
\varepsilon_{\mathbf{k}}^{\beta_1} = \frac{(k_x - \pi/a)^2}{2m_x} + \frac{k_y^2}{2m_y} - \mu \quad \varepsilon_{\mathbf{k}}^{\beta_2} = \frac{k_x^2}{2m_x} + \frac{(k_y - \pi/a)^2}{2m_x} - \mu 
\]

- Fermi sheets for electron-hole doped case ($\text{Fe}^{2+} \Rightarrow 3d^6$)
III. Theoretical background

A. Spin fluctuation pairing

• Historical: ferromagnetic spin fluctuations
  • Transition metals near ferromagnetism
  • Exchange of “paramagnons”
III. Theoretical background

A. Spin fluctuation pairing

- **Antiferromagnetic spin fluctuations**
  - Susceptibility strongly peaked near $Q$
  - Singlet interaction (always repulsive)
    $$\Gamma_s(q) = \frac{3}{2} U^2 \frac{\chi_0(q)}{1 - U \chi_0(q)}$$
  - Self-consistent BCS gap equation
    $$\Delta_k = - \sum_{k'} \Gamma_s(k, k') \frac{\Delta'_{k'}}{2E'_k} \tanh \left( \frac{E'_k}{2T} \right)$$
  - Solution possible for
    $$\Delta_k = -\Delta_{k+Q}$$
  - In cuprates, $\chi$ is peaked at $Q = (\pi, \pi)$
    $$\Delta^{d,s}_k = \Delta_0 (\cos(k_x a) \mp \cos(k_y a))$$
  - In pnictides, $\chi$ is peaked at $Q = (\pi, 0) \Rightarrow s_\pm$ pairing
III. Theoretical background

A. Spin fluctuation pairing

- Spin fluctuation pairing in multi-orbital systems

\[
H_0 = \sum_{\mathbf{k}, \sigma, i = \alpha_1, \alpha_2, \beta_1, \beta_2} \varepsilon_{\mathbf{k}}^i c_{i\mathbf{k}\sigma}^{\dagger} c_{i\mathbf{k}\sigma}
\]

\[
H = H_0 + U \sum_{i, \ell} n_{i\ell\uparrow} n_{i\ell\downarrow} + U' \sum_{i, \ell' < \ell} n_{i\ell} n_{i\ell'} + J \sum_{i, \ell' < \ell} \sum_{\sigma, \sigma'} c_{i\ell\sigma}^{\dagger} c_{i\ell'\sigma'}^{\dagger} c_{i\ell\sigma'} c_{i\ell'\sigma}
\]

\[
+ J' \sum_{i, \ell' \neq \ell} c_{i\ell\uparrow}^{\dagger} c_{i\ell\downarrow}^{\dagger} c_{i\ell'\downarrow} c_{i\ell'\uparrow}
\]

- Effective pair scattering vertex between bands \(i\) and \(j\) in the singlet channel

\[
\Gamma_{ij}(\mathbf{k}, \mathbf{k}') = \text{Re} \left[ \sum_{\ell_1 \ell_2 \ell_3 \ell_4} a_{\nu_i}^{\ell_2, *}(\mathbf{k}) a_{\nu_i}^{\ell_3, *}(\mathbf{-k}) \Gamma_{\ell_1, \ell_2, \ell_3, \ell_4}(\mathbf{k}, \mathbf{k}', \omega = 0) a_{\nu_j}^{\ell_1}(\mathbf{k}') a_{\nu_j}^{\ell_4}(\mathbf{-k}') \right]
\]

\[
a_{\nu}(\mathbf{k}) = \langle \ell | \nu k \rangle \quad \mathbf{k} \in C_i \quad \mathbf{k}' \in C_j
\]

- Orbital vertex functions

\[
\Gamma_{\ell_1 \ell_2 \ell_3 \ell_4}(\mathbf{k}, \mathbf{k}', \omega) = \left[ \frac{3}{2} U^s \chi_1^{\text{RPA}}(\mathbf{k} - \mathbf{k}', \omega) U^s + \frac{1}{2} U^s \right] \left[ -\frac{1}{2} U^c \chi_0^{\text{RPA}}(\mathbf{k} - \mathbf{k}', \omega) U^c + \frac{1}{2} U^c \right]_{\ell_1 \ell_2 \ell_3 \ell_4}
\]

Charge/orbital fluctuations

Spin fluctuations
III. Theoretical background

A. Spin fluctuation pairing

- Results of microscopic theory
  - Gap symmetry and structure
    \[ \Delta(k) = \Delta g(k) \]
  - Orbital vertex functions
    \[
    \lambda[g(k)] = -\frac{\sum_{ij} \int_{C_i} \frac{dk_{\parallel}}{v_F(k)} \int_{C_j} \frac{dk'_{\parallel}}{v_F(k')} g(k) \Gamma_{ij}(k, k') g(k')}{(2\pi)^2 \sum_i \int_{C_i} \frac{dk_{\parallel}}{v_F(k)} g^2(k)}
    \]
    \[ k \in C_i \quad k' \in C_j \]
    \[ v_{F,\nu}(k) = |\nabla_k E_{\nu}(k)| \]
III. Theoretical background

A. Spin fluctuation pairing

• Physical origins of anisotropy of pair state and node formation

  • Intra-orbital pairing between $\alpha$ and $\beta$ Fermi sheets $\Rightarrow$ favors $s_{\pm}$

  • Sub-leading inter-orbital between $\beta_1$ and $\beta_2$ Fermi sheets $\Rightarrow$ favors nodes $\Rightarrow$ frustrates $s_{\pm}$

  • Hole doping ($n < 6$) $\Rightarrow$ appearance of $\gamma$ pocket

  • Appearance of $\gamma$ pocket $\Rightarrow$ $\beta_1$-$\beta_2$ scattering causes weaker $s_{\pm}$ frustration

  • Height of Arsenic above Fe-plane $\Rightarrow$ appearance of $\gamma$ pocket and isotropy of $s_{\pm}$ state in 1111 family
III. Theoretical background

B. Alternative approaches

- **Excitonic Superconductivity**
  - Proposed by Little and Ginzburg
  - Exciton + phonon $\Rightarrow$ CDW $\Rightarrow$ suppress superconductivity

- **Orbital fluctuations**
  - Ordering of Fe $3d$ orbitals possible
  - Recall Hamiltonian

$$H = H_0 + \tilde{U} \sum_{i,\ell} n_{i\ell\uparrow} n_{i\ell\downarrow} + \tilde{U}' \sum_{i,\ell' < \ell} n_{i\ell} n_{i\ell'} + \tilde{J} \sum_{i,\ell' < \ell} \sum_{\sigma,\sigma'} c_{i\ell\sigma}^\dagger c_{i\ell'\sigma'}^\dagger c_{i\ell\sigma'} c_{i\ell'\sigma}$$

  - Cannot have $\tilde{U}' > \tilde{U}$, $\tilde{J}' > \tilde{J}$ for purely electronic interactions
V. Gap structure

A. Does the gap in FeBS change sign?

- Spin-resonance peak
  - Neutron scattering $\rightarrow$ dynamical spin susceptibility

$$\chi_s(q, \omega) = \frac{\chi_0(q, \omega)}{1 - U_s \chi_0(q, \omega)}$$
V. Gap structure

A. Does the gap in FeBS change sign?

- Josephson junctions
  - $d$-wave symmetry of cuprates confirmed by Josephson effect
  - Phase shift of $\pi$ between orthogonal planes
  - Unfortunately pnictides don’t have spatial anisotropy
  - Phase difference across electron and hole Fermi sheets for $s^\pm$ pairing
  - Solution $\Rightarrow$ Measure Josephson effect across epitaxially grown interface between electron- and hole-doped pnictide
V. Gap structure

A. Does the gap in FeBS change sign?

- Quasiparticle interference $\propto u_k u_{k'} + v_k v_{k'}$

\[ |v_k|^2 = 1 - |u_k|^2 = \frac{1}{2} \left( 1 - \frac{\epsilon_k}{E_k} \right)^{1/2} \quad E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2} \]

- QPI in the Cuprates

\[ \epsilon_k = -2\chi_0 (\cos(k_x) + \cos(k_y)) \]
\[ \Delta_k = 2\Delta_0 (\cos(k_x) - \cos(k_y)) \]

- QPI in the Pnictides $\Rightarrow$ no “hot spots”

- Bragg vs. QPI?
V. Gap structure

A. Does the gap in FeBS change sign?

- Coexistence of magnetism and superconductivity
  - Co-doped BaFe$_2$As$_2$ $\Rightarrow$ Microscopic coexistence of weak antiferromagnetism and superconductivity
  - $s_{\pm}$ or $s_{++}$ states coexists with SDW
  - In-plane thermal conductivity experiment $\Rightarrow$ no $c$-axis nodal lines
  - $s_{++}$ pairing $\Rightarrow$ SDW-induced BZ band folding $\Rightarrow$ $c$-axis line nodes $\Rightarrow$ $s_{++}$ pairing cannot coexist with SDW
V. Gap structure

B. Evidence for low-energy subgap excitations

- Evidence for very low energy excitations consistent with gap nodes
- Bulk probes provide consistent picture of the evolution of the low-energy quasiparticle density across the phase diagram
V. Gap structure

B. Evidence for low-energy subgap excitations

- Penetration depth
  - $\Delta \lambda \propto \exp(-\Delta_{\text{min}}/T) \Rightarrow$ fully gapped (optimally doped Ba$_{1-x}$K$_x$Fe$_2$As$_2$)
  - $\Delta \lambda \propto T \Rightarrow$ line nodes (1111 family)
  - $\Delta \lambda \propto T^2 \Rightarrow$ disorder (122 family)
V. Gap structure

B. Evidence for low-energy subgap excitations

- Specific heat
  - Nodes $\Rightarrow$ Volovik effect $\Rightarrow$ $C/T \propto H^{1/2}$
  - Fully gapped $\Rightarrow$ vortex cores states $\Rightarrow$ $C/T \propto H$
  - Fe(Te,Se) specific heat oscillations
V. Gap structure

B. Evidence for low-energy subgap excitations

• The ARPES “paradox”
  • The most direct probe of gap structure \(\Rightarrow\) proved \(d\)-wave pairing in cuprates
  • No ARPES study has seen nodes in the gap

• Possible explanations of the paradox
  • *Surface electronic reconstruction*
    • Surface DFT on \(\text{BaFe}_2\text{As}_2\) \(\Rightarrow\) additional \(d_{xy}\) pocket
    • \(d_{xy}\) pocket stabilizes isotropic pair state
  • *Surface depairing*
    • Surface roughness \(\Rightarrow\) in-plane intraband scattering \(\Rightarrow\) destroys gap anisotropy

• *Resolution issues*
Summary

- **Bardeen-Cooper Schrieffer (conventional) superconductors**
  
  - Discovered in 1911 by Kamerlingh-Onnes
  
  - Fully gapped Bogoliubov quasiparticle spectrum
  
  - Important effects
    - Vanishing resistivity
    - Meissner effect (London penetration depth)
    - Coherence effects (coherence length)

- **Heavy-fermion superconductors**
  
  - Discovered by Steglich *et al.* in 1979
  
  - Key ingredients
    - **Lattice** of \( f \)-electrons
    - Conduction electrons
  
  - Multiple superconducting phases
Summary

- **Electronic structure**
  - Relevant physics confined to 2D
  - The “t-J” model
    \[ H = P \left[ - \sum_{\langle ij \rangle, \sigma} t_{ij} c_{i\sigma}^\dagger c_{i\sigma} + J \sum_{\langle ij \rangle} \left( S_i \cdot S_j - \frac{1}{2} n_i n_j \right) \right] P \]
  - Universal phase diagram

- **Phenomenology of the cuprates**
  - Experimental signatures of the pseudogap phase
  - Nodal quasiparticles

- **Slave bosons**
  - Slave fermions and bosons
  - U(1) & SU(2) gauge theory
Summary

- **Iron-based (pnictide) superconductors**
  - Discovered in 2008 by Kamihara
  - Physics confined to 2D like cuprates
  - Pseudogap replaced by “nematic phase”

- **Gap structure**
  - Theory and some experiments $\rightarrow s^\pm$
  - Contradictory experimental evidence
    - ARPES
    - Specific heat
    - Penetration depth
Thanks for listening!

Questions?
Future Topics

• Heavy fermion materials
  • Not limited to superconductivity
  • e.g. Kondo topological insulators

• Exotic topics motivated by the cuprates
  • Gauge theories and confinement physics
  • Quantum critical point (QCP)

• Competing interpretations of cuprate high-Tc?
  • Stripe or no stripe?
  • Is slave boson picture nonsense?

How to detect fluctuating stripes in the high-temperature superconductors

S. A. Kivelson and I. P. Bindloss
Department of Physics, University of California at Los Angeles, Los Angeles, California 90095, USA

E. Fradkin
Department of Physics, University of Illinois, Urbana, Illinois 61801-3080, USA

V. Oganesyyan
Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

J. M. Tranquada
Physics Department, Brookhaven National Laboratory, Upton, New York 11973-5000, USA

A. Kapitulnik and C. Howald
Department of Physics, Stanford University, Stanford, California 94305-4045, USA

(Published 8 October 2003)

This article discusses fluctuating order in a quantum disordered phase proximate to a quantum critical point, with particular emphasis on fluctuating stripe order. Optimal strategies are derived for extracting information concerning such local order from experiments, with emphasis on neutron scattering and scanning tunneling microscopy. These ideas are tested by application to two model systems—an exactly solvable one-dimensional (1D) electron gas with an impurity, and a weakly interacting 2D electron gas. Experiments on the cuprate high-temperature superconductors which can be analyzed using these strategies are extensively reviewed. The authors adduce evidence that stripe correlations are widespread in the cuprates. They compare and contrast the advantages of two limiting perspectives on the high-temperature superconductor: weak coupling, in which correlation effects are treated as a perturbation on an underlying metallic (although renormalized) Fermi-liquid state, and strong coupling, in which the magnetism is associated with well-defined localized spins, and stripes are viewed as a form of micro phase separation. The authors present quantitative indicators that the latter view better accounts for the observed stripe phenomena in the cuprates.